14

Sample collection

Category I: Linear systems	**3**
Category II: Controller design	**11**
Category III: Simulation	**22**
Category IV: Fuzzy logic	**41**
Category V: Fuzzy control	**48**
Category VI: Measurement technology	**56**
Category VII: Digital technology	**64**
Category VIII: System identification	**71**

Intent of this sample collection

This sample collection contains a lot of various Problems which are divided into the following categories:

I. Linear systems

II. Controller design

III. Simulation

IV. Fuzzy logic

V. Fuzzy control

VI. Measurement

VII. Digital circuits

VIII. System identification

All Problems can be solved by using WinFACT, sometimes in combination with a little "hand work". We tried to cover all technical topics to let each user meet "his" favourite topic. The intent of this collection is on one hand to present an overview of WinFACT's components and features, on the other hand to help unexperienced users to get familiar with WinFACT. Those users working in the field of teaching might use this collection as a pool of Problems and ideas for the formulation of own exercises.

All Problem contains the Problem description itself, a solution sketch containing all necessary hints and in most cases a corresponding system file which illustrates the solution.

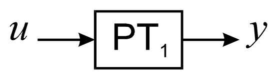
Remark: Most screenshots are captured with older versions of WinFACT. Nevertheless the corresponding files can be used with the current release.

Category I: Linear systems

Problem I.1: Step response of a PT1-element

Problem specification: Given a PT1-element with input $u(t)$ and output $y(t)$ and the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{1+Ts}$$

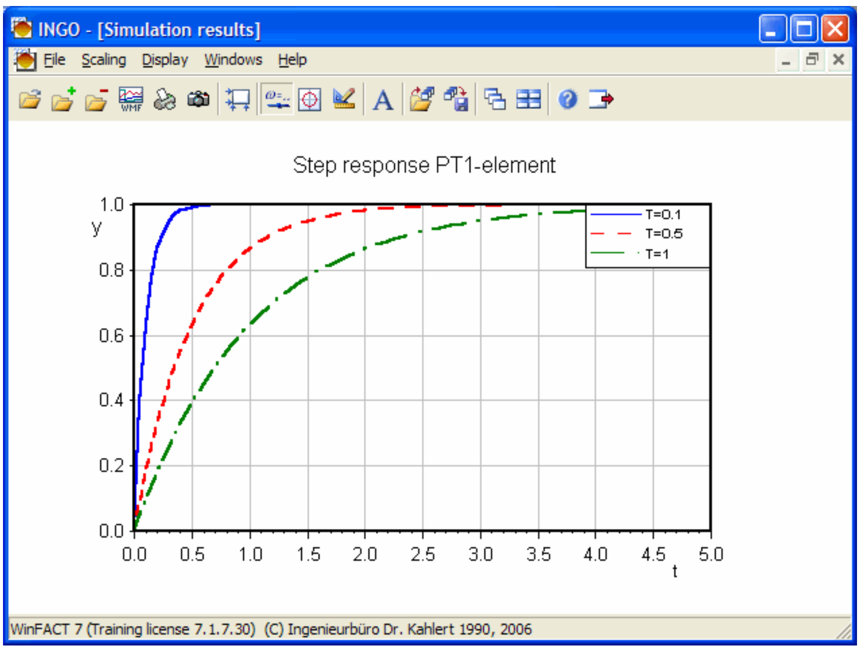


resp. the differential equation

$$T\dot{y} + y = Ku$$

Determine the step response of the system for $K = 1$ and time constants of T = 0.1, 0.5 and 1. Select a simulation length of 5.

Solution: This Problem demonstrates the influence of the time constant on the dynamic behaviour of the system. The solution can be obtained by using LISA or BORIS. The single step responses can be saved in SIM-files and compared with INGO (see screenshot below).



 © Ingenieurbüro Dr. Kahlert 1991-2006

Problem I.2: Frequency response of a PT1-element

Problem specification: Given a PT1-element with input $u(t)$ and output $y(t)$ and the transfer function

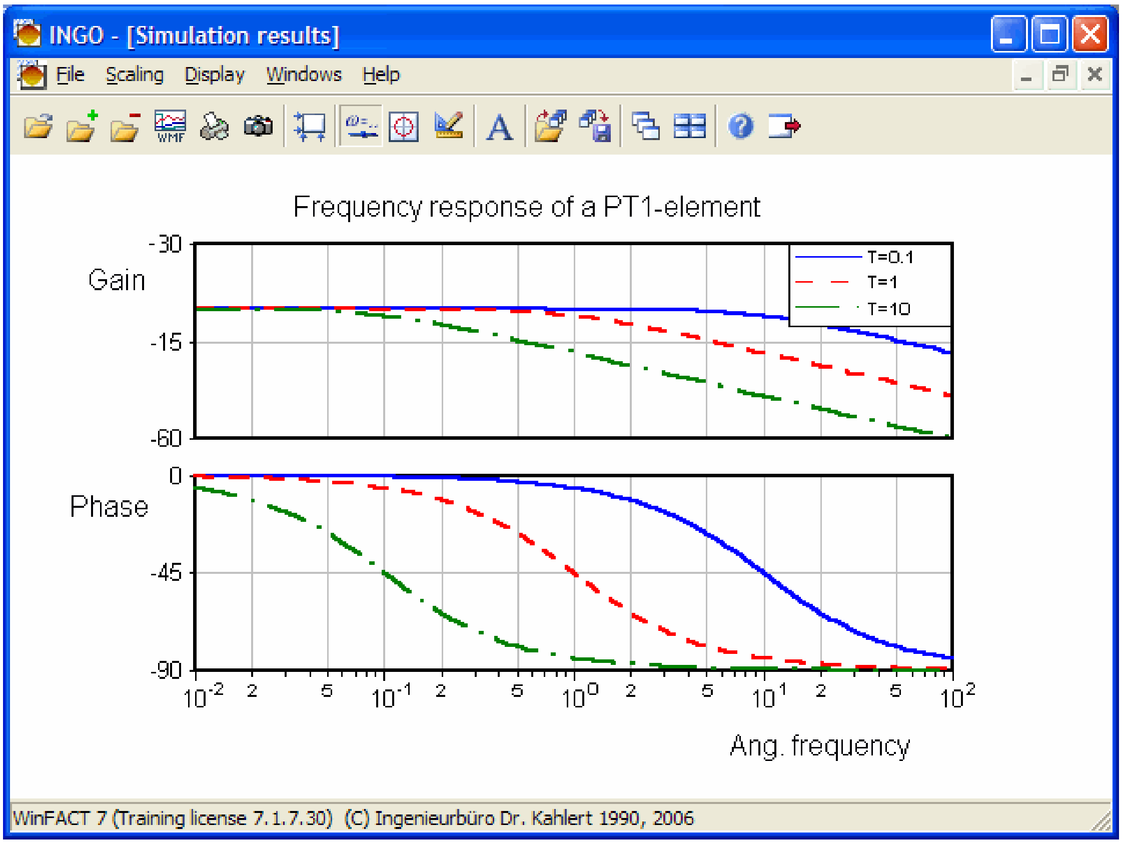
$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{1+Ts}$$

resp. the differential equation

$$T\dot{y} + y = Ku$$

Determine the frequency response of the system for $K = 1$ and time constants of $T = 0.1$, 1 and 10 for a frequency range of $0.01 \le \omega \le 100$.

Solution: This Problem demonstrates the influence of the time constant resp. eigenfrequency on the frequency response of the system. The solution can be obtained by using LISA or BORIS. The single step responses can be saved in SIM-files and compared with INGO (see screenshot below).



Problem I.3: Step response of a PT_2-element

Problem specification: Given a PT2-element with input $u(t)$ and output $y(t)$ and the transfer function

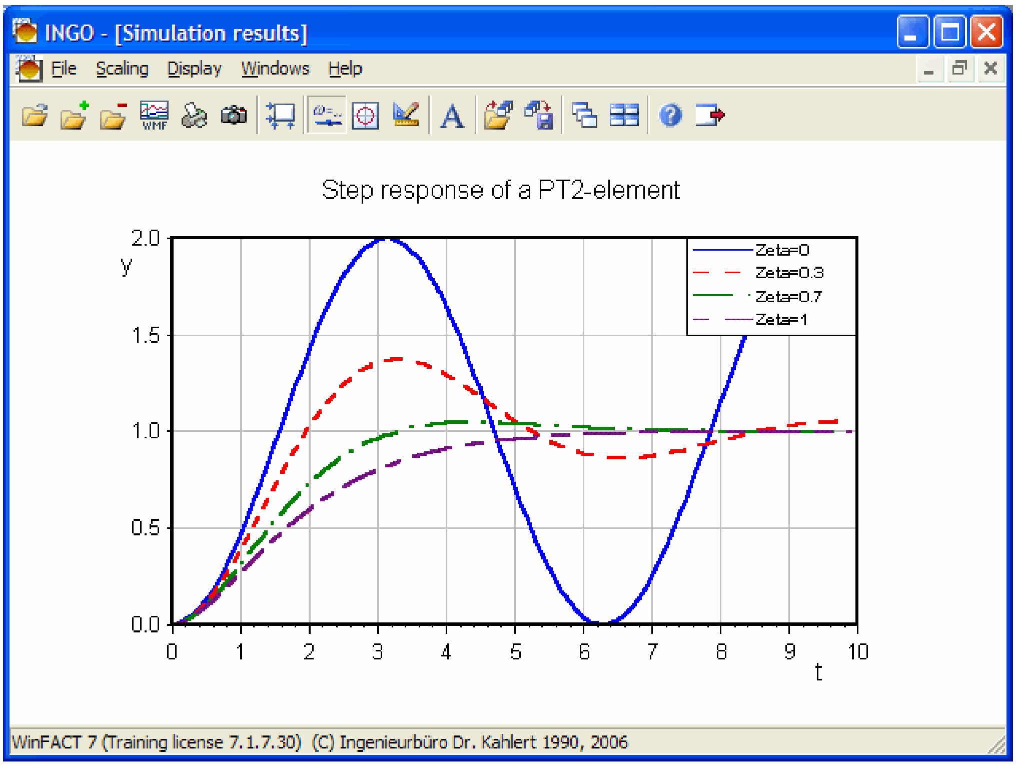
$$G(s) = \frac{K}{\left(\frac{s}{\omega_n}\right)^2 + 2\frac{\zeta}{\omega_n}s + 1}$$

resp. the differential equation

$$\frac{1}{\omega_n^2}\ddot{y} + 2\frac{\zeta}{\omega_n}\dot{y} + y = Ku$$

Determine the step response of the system for $K = \omega_n = 1$ and damping coefficients of $\zeta = 0,\ 0.3,\ 0.7,\ 1$. Select a simulation length of 10.

Solution: This Problem demonstrates the influence of the damping coefficient ζ on the dynamic behaviour of the system. The solution can be obtained by using LISA or BORIS. The single step responses can be saved in SIM-files and compared with INGO (see screenshot below).



 © Ingenieurbüro Dr. Kahlert 1991-2006

Problem I.4: Frequency response of a PT_2-element

Problem specification: Given a PT2-element with input $u(t)$ and output $y(t)$ and the transfer function

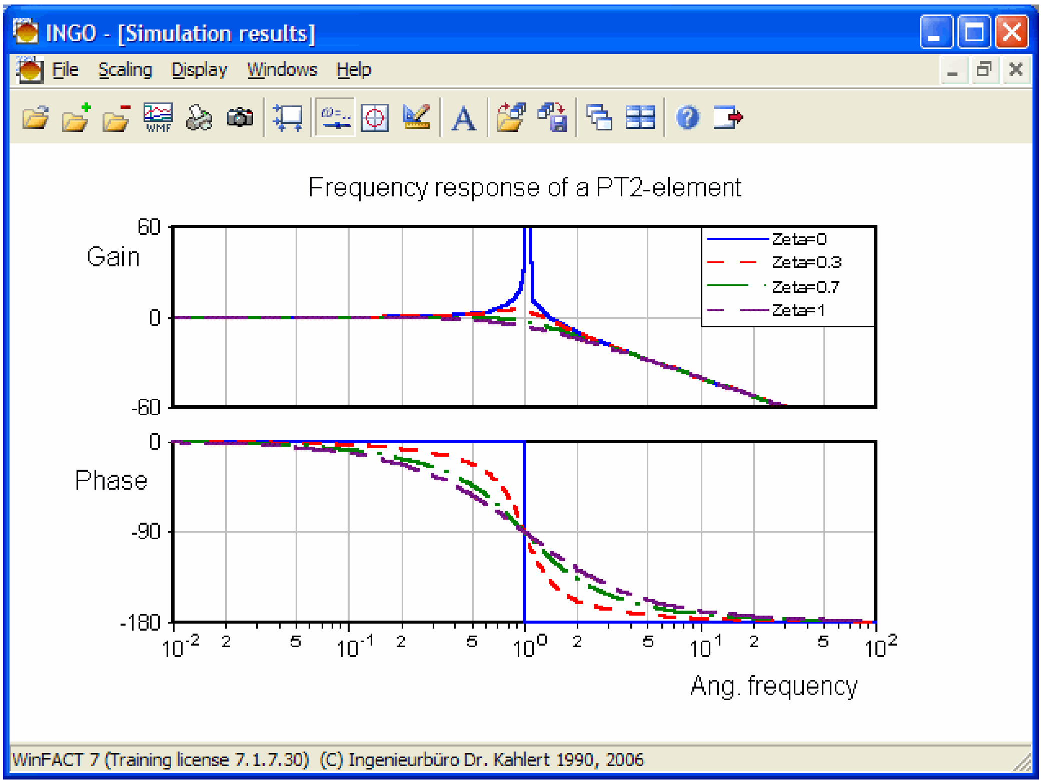
$$G(s) = \frac{K}{\left(\frac{s}{\omega_n}\right)^2 + 2\frac{\zeta}{\omega_n}s + 1}$$

resp. the differential equation

$$\frac{1}{\omega_n^2}\ddot{y} + 2\frac{\zeta}{\omega_n}\dot{y} + y = Ku$$

Determine the frequency response of the system for $K = \omega_n = 1$ and damping coefficients of $\zeta = 0,\ 0.3,\ 0.7,\ 1$ for a frequency range of $0.01 \le \omega \le 100$.

Solution: This Problem demonstrates the influence of the damping coefficient ζ on the frequency response of the system. The solution can be obtained by using LISA or BORIS. The single step responses can be saved in SIM-files and compared with INGO (see screenshot below).



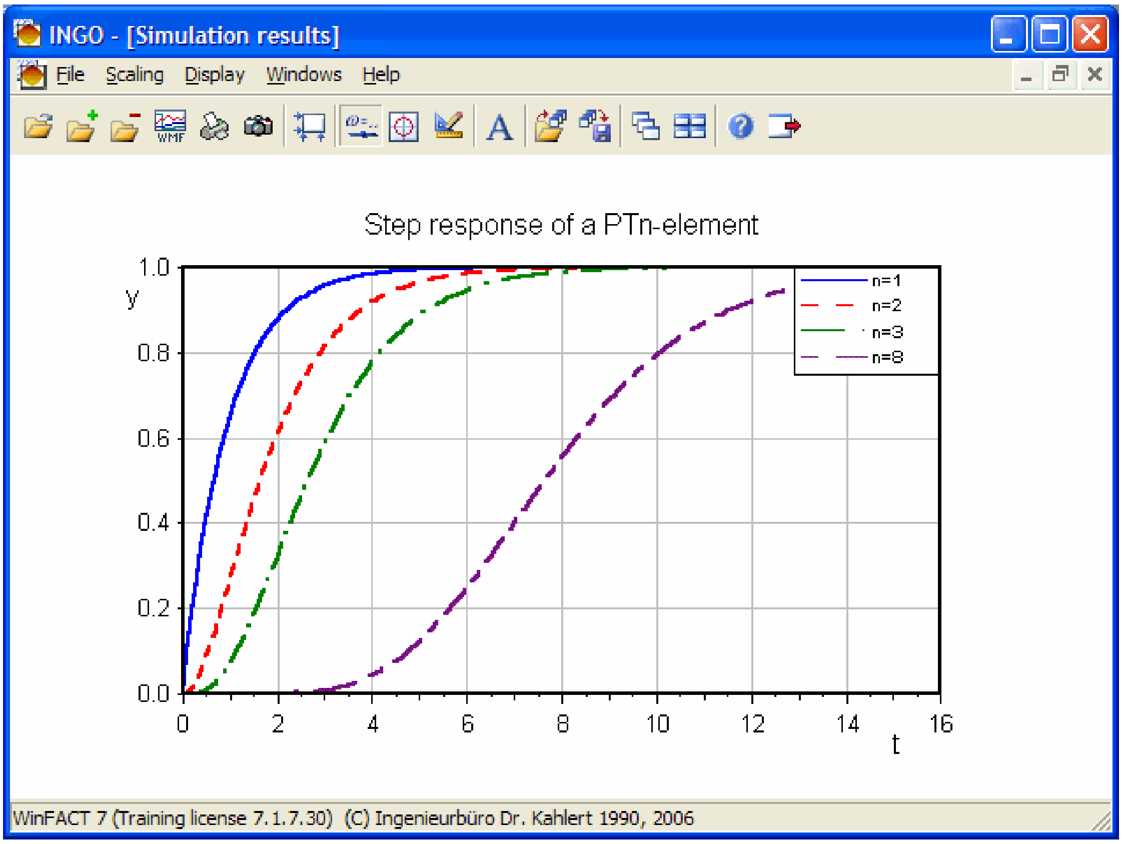
Problem I.5: Step response of a PTn-element

Problem specification: Given a PT_n-element with n identical time constants and the transfer function

$$G(s) = \frac{1}{(1+s)^n}.$$

Determine the step response of the system for n = 1, 2, 3 and 8. Select a simulation length of 15.

Solution: This Problem demonstrates the influence of the system order n on the dynamic behaviour of the system. The solution can be obtained by using LISA or BORIS (recommended). The single step responses can be saved in SIM-files and compared with INGO (see screenshot below).



The results show that the system reacts slower with an increasing value of n. For n = 8 the system shows a dead time-similar behaviour.

Related files:

PTN.BSY

 © Ingenieurbüro Dr. Kahlert 1991-2006

Problem I.6: All-pass system

Problem specifcation: Given a second-order system with the transfer function

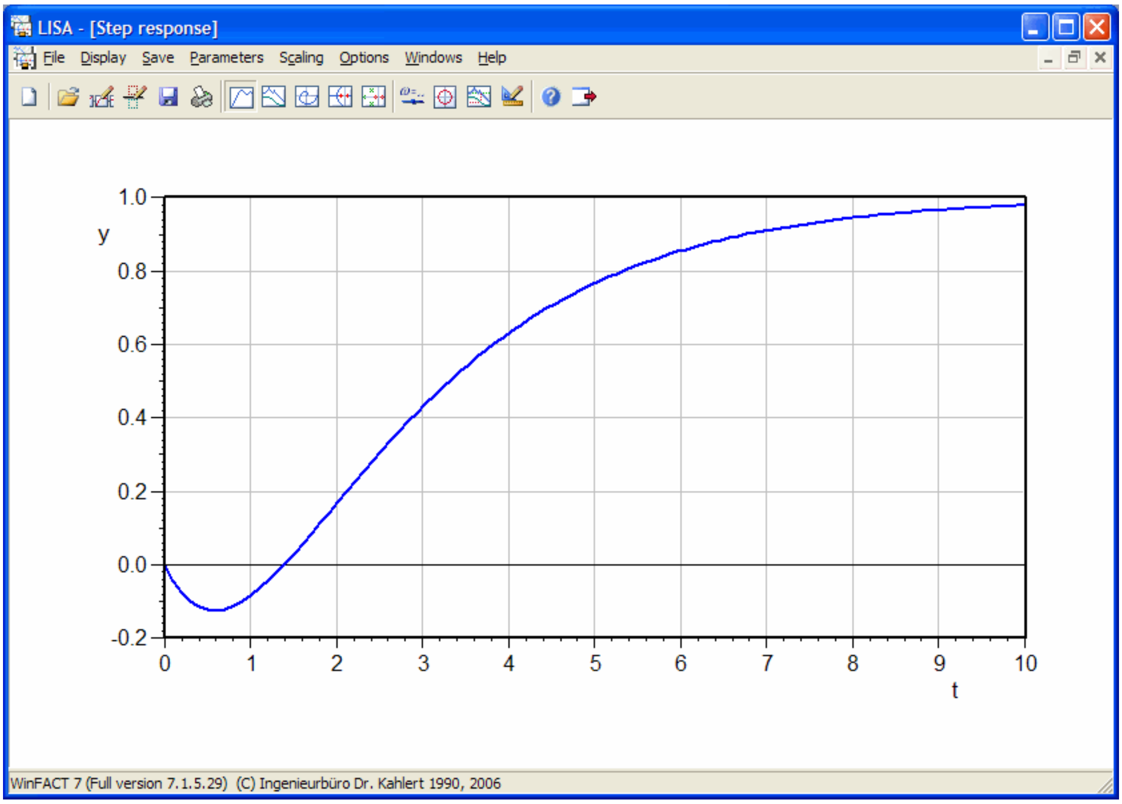
$$G(s) = \frac{1-s}{(1+s)(1+2s)}.$$

Determine poles and zeros of the system. Simulate the corresponding step response up to a simulation time of 10.

Solution: The determination of poles and zeros as well as the simulation itself can be executed with LISA. The results for poles and zeros are:

$$n_1 = 1 \qquad p_1 = -1 \quad p_2 = -0.5$$

The system has a zero in the right half-plane and so all-pass characteristic. The simulation delivers the step response shown below which shows the all-pass typical undershoot at the beginning of the simulation.



Related files:

ALLPASS.UFK

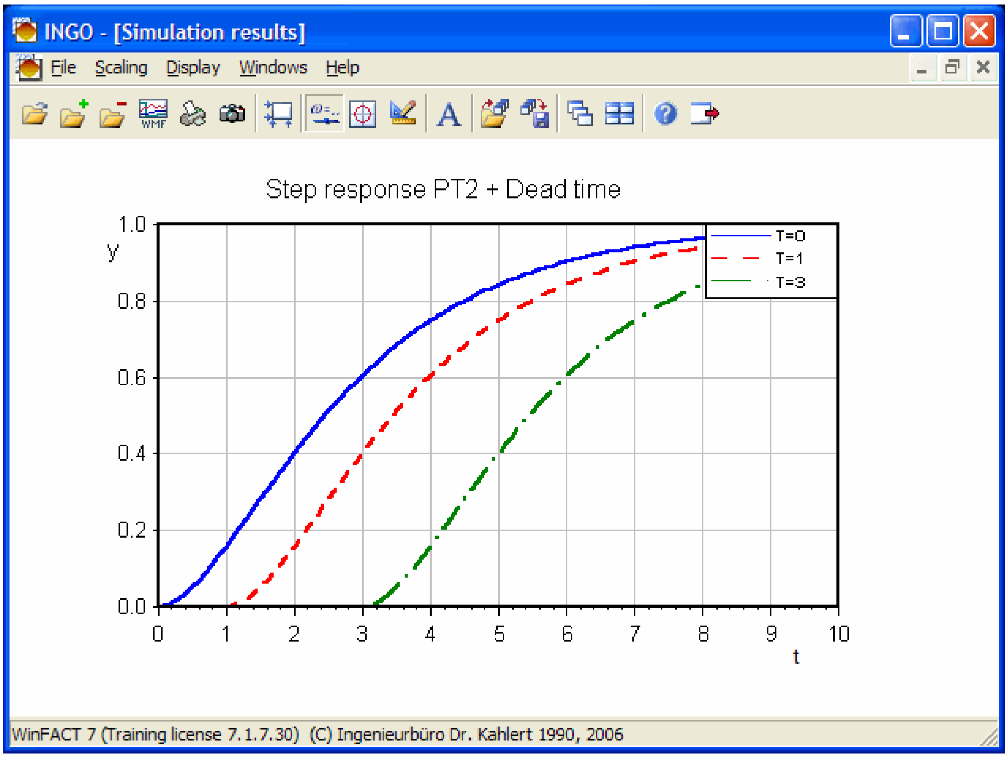
Problem I.7: System with dead time

Problem specification: Given a system with the transfer function

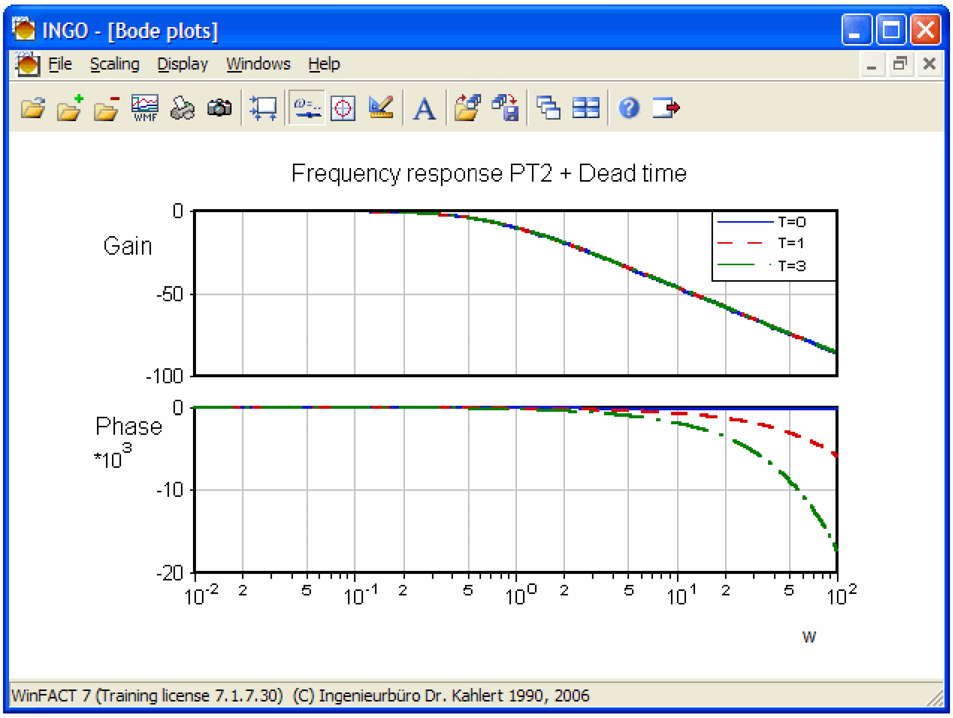
$$G(s) = \frac{1}{(1+s)(1+2s)} e^{-Ts}.$$

Determine step and frequency response of the system for dead times of $T = 0$, 1 und 3. Select a simulation length of 10 resp. a frequency range of $0.01 \le \omega \le 100$.

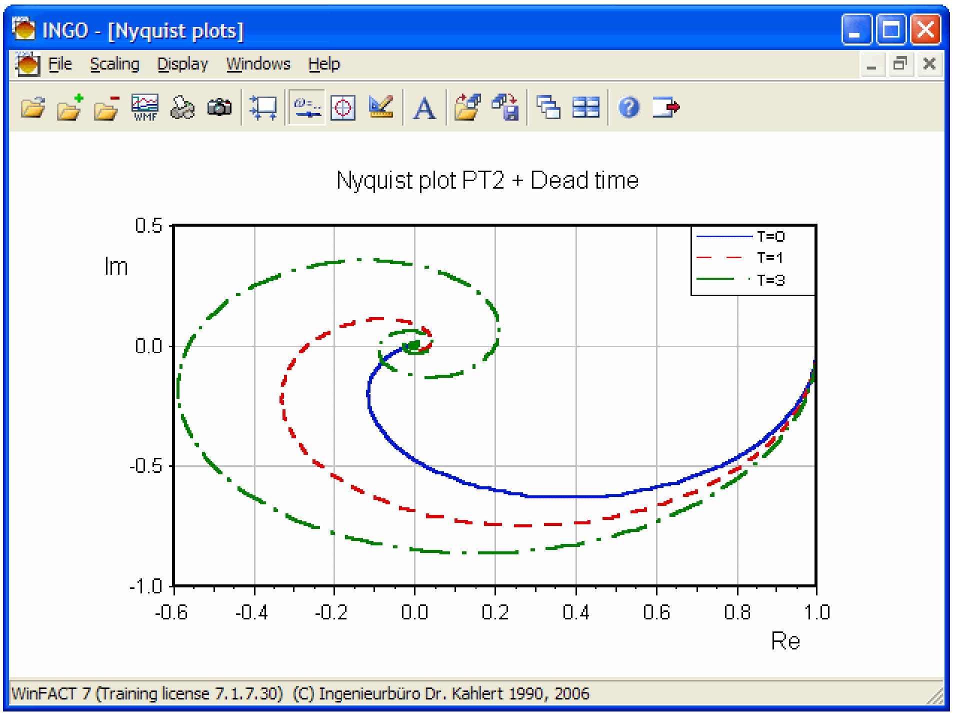
Solution: This Problem demonstrates the influence of the dead time on the dynamic behaviour of the system. The solution can be obtained by using LISA or BORIS. The results can be saved and compared with INGO. For step responses and Bode plots you get the following results. A greater dead time leads to a step response that starts "later". The dead time has no influence on the gain of the Bode plot but effects a monotone decrease of the phase.



 © Ingenieurbüro Dr. Kahlert 1991-2006



For the Nyquist plot we get the result shown in the screenshot below:



In this representation mode the dead time causes the characteristical circuiting of the curve around the origin of the complex plane.

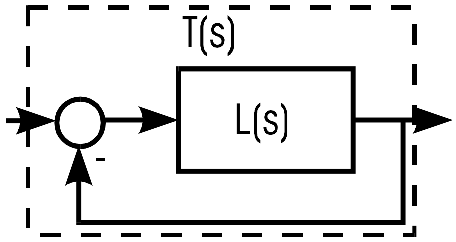
Related files:

TOTZEIT.UFK

Category II: Controller design

Problem II.1: Correlation between phase margin Φ_r and overshoot M_p

Problem specification: Given the following closed-loop system:



It is

$$L(s) = \frac{K}{s^2 + s + 1}$$

the transfer function of the open-loop system. Determine the correlation between the phase margin Φ_r of the open-loop system $L(j\omega)$ and the corresponding overshoot M_p of the step response of the closed-loop system

$$T(s) = \frac{L(s)}{1 + L(s)}.$$

by selecting different values for the gain *K*. Represent the correlation graphically.

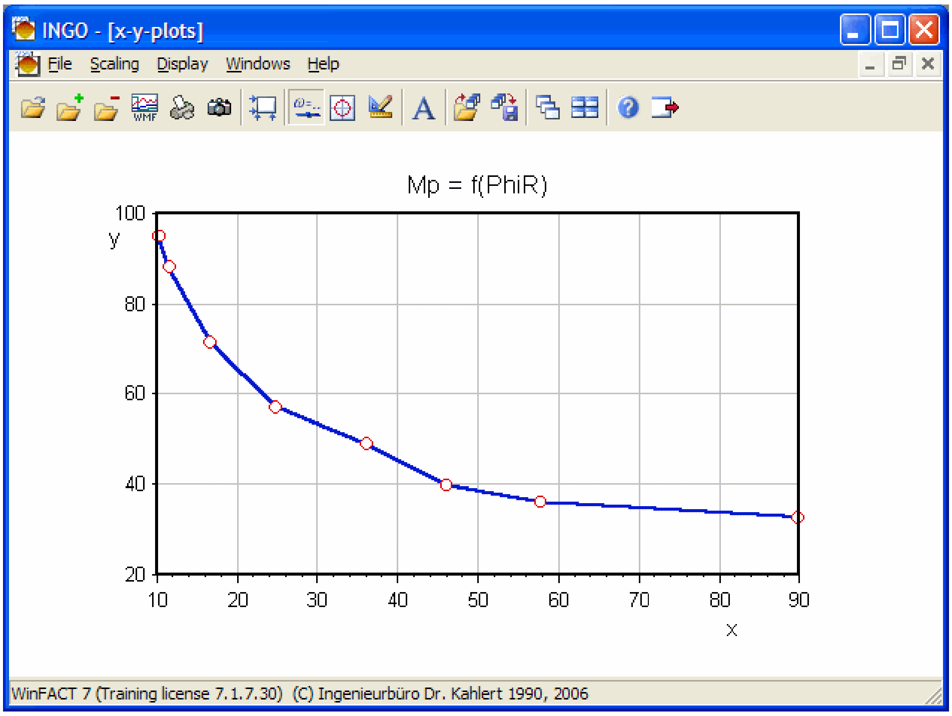
Solution: For getting the solution module RESY can be used. We get the following results:

K	Φ_r	M_p
1	89.8	32.6
1.5	57.8	36

 © Ingenieurbüro Dr. Kahlert 1991-2006

2	46.1	39.7
3	36.2	48.9
5	24.9	57
10	16.7	71.5
20	11.6	88
30	10.3	95

The function $M_p = f(\Phi_r)$ can be presented graphically e. g. by using module INGO:



Related files: PHIRMP.UFK
PHIRMP.XY

Problem II.2: Controller design by "rules of thumb"

Problem specification: Given a plant with the transfer function

$$G(s) = \frac{0.149}{s^2 + 1.132s + 0.1772}.$$

Determine the step response up to a time of 25 first and specify gain K_S, delay time T_u and transition time T_g. Afterwards based on these results design a PID-controller by the rules of thumb from Samal which are as follows:

Controller with overshoot of output variable:

$$K_R = 0.95 \frac{T_g}{K_S T_u} \quad T_N = 1.35 T_g \quad T_V = 0.47 T_u$$

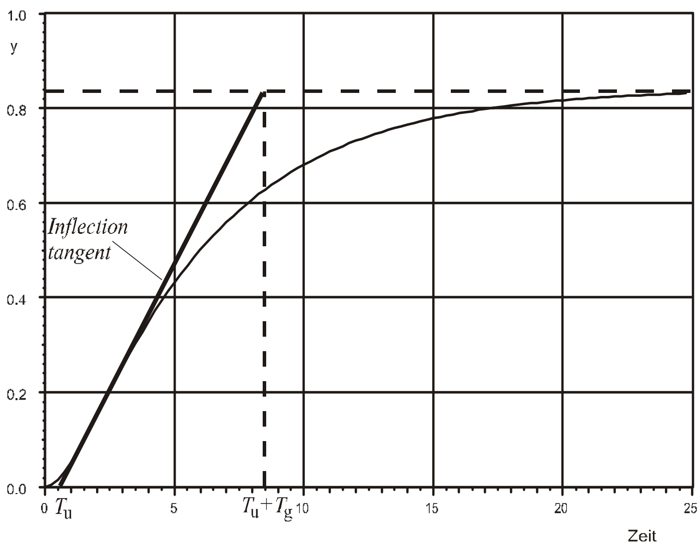
Controller without overshoot of output variable:

$$K_R = 0.59 \frac{T_g}{K_S T_u} \quad T_N = T_g \quad T_V = 0.5 T_u$$

Simulate the step responses of both closed-loop systems and compare them.

Solution: The determination of the step response of the plant can be done by using LISA or BORIS. The characteristic values can be specified by hand (see figure below). We get the following results:

$$K_S \approx 0.85 \quad T_u \approx 0.55 \quad T_g \approx 8$$



This leads to the following controller parameters:

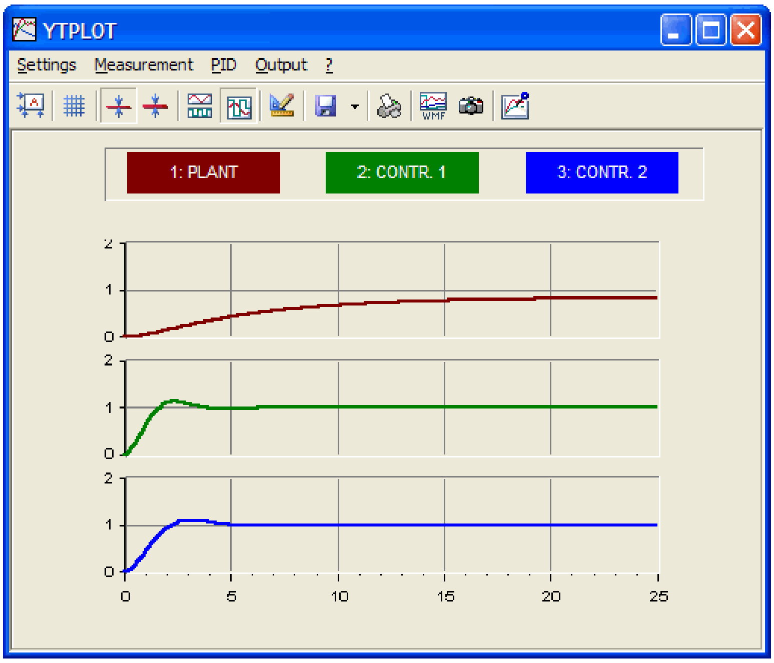
Controller with overshoot of output variable:

$$K_R = 16.3 \quad T_N = 10.8 \quad T_V = 0.26$$

Controller without overshoot of output variable:

$$K_R = 10.1 \quad T_N = 8 \quad T_V = 0.28$$

Using BORIS for the simulation of the closed-loop systems we get the following results:



We recognize that the dynamics of the closed-loop system is improved compared to the plant itself (top curve). Both controllers lead to an overshoot of the controlled variable; this overshoot is less if the controller designed for a step response without overshoot is used. Because this controller has a smaller gain than the first one (10.1 compared to 16.3) the closed-loop system has a greater rise and settling time compared to the loop with the first controller.

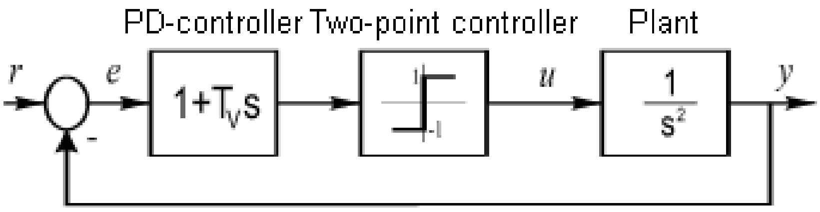
Related files: EINSTELL.UFK
EINSTELL.BSY

Problem II.3: Nonlinear control (Sliding mode controller)

Problem specification: Given a plant with the transfer function

$$G(s) = \frac{1}{s^2} \quad \text{(double integrator)}.$$

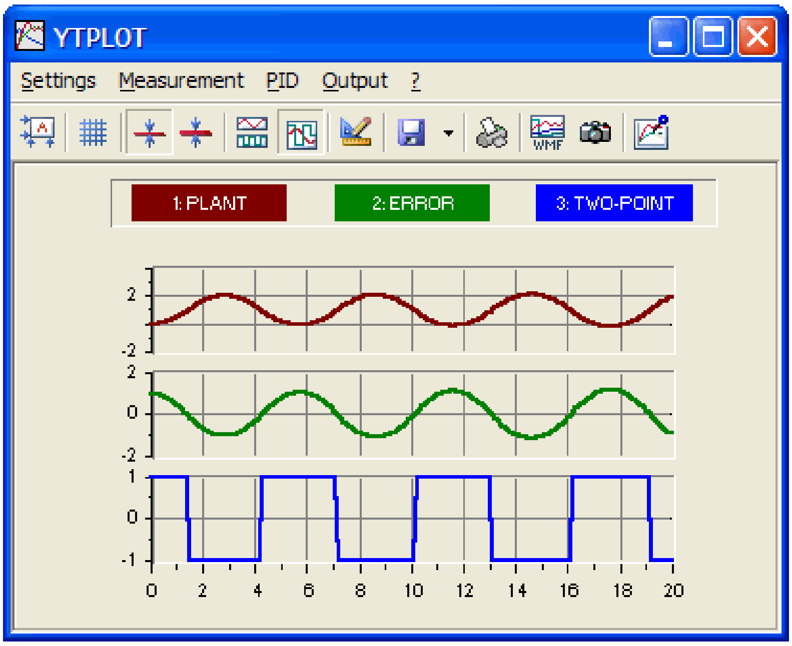
For stabilization of the plant a serial connection of a PD-controller and a two-point controller is to be applied:



Determine the rate time T_V of the PD-controller in such a way that a step-sized reference signal is adapted as fast as possible but the manipulated variable does not take too high values. For simulation of the system choose the Runge-Kutta method with a step size of $\Delta T = 0.01$ and a simulation length of 20.

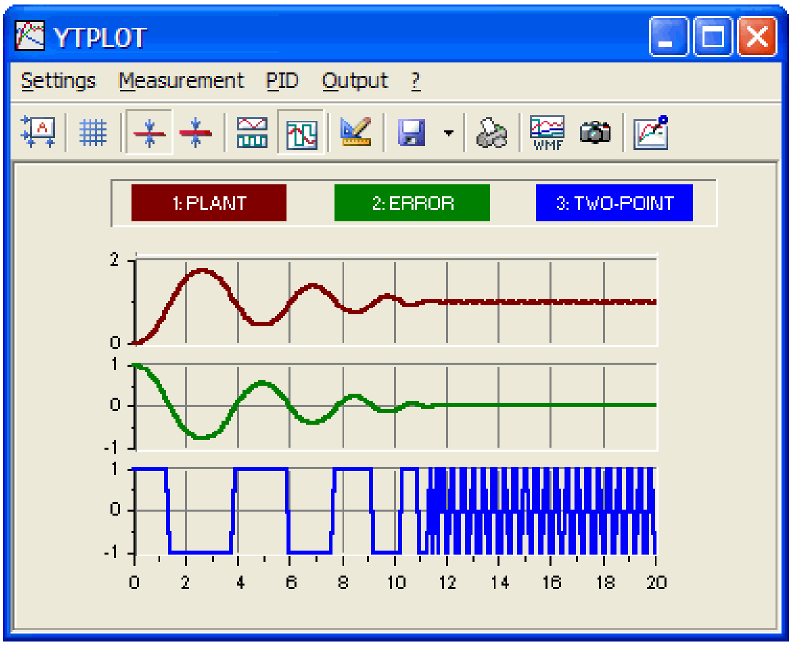
Solution: This controller is a so-called *sliding mode controller* which transfers the system trajectory into a sliding mode after an oscillation at the beginning of the control process; this sliding mode leads to the stationary state after a short time. The larger the rate time T_V is chosen the faster the sliding mode is reached but the more time the sliding mode needs. In the sliding mode the two-point controller switches between both operating points permanently; thus the actuator is loaded very hard. So a compromise has to be found for the rate time leading to a fast response of the system combined with a limited load of the actuator.

The screenshots below show the response of the output variable of the plant (top), the error signal (middle) and the manipulated variable (bottom) for different values of T_V.

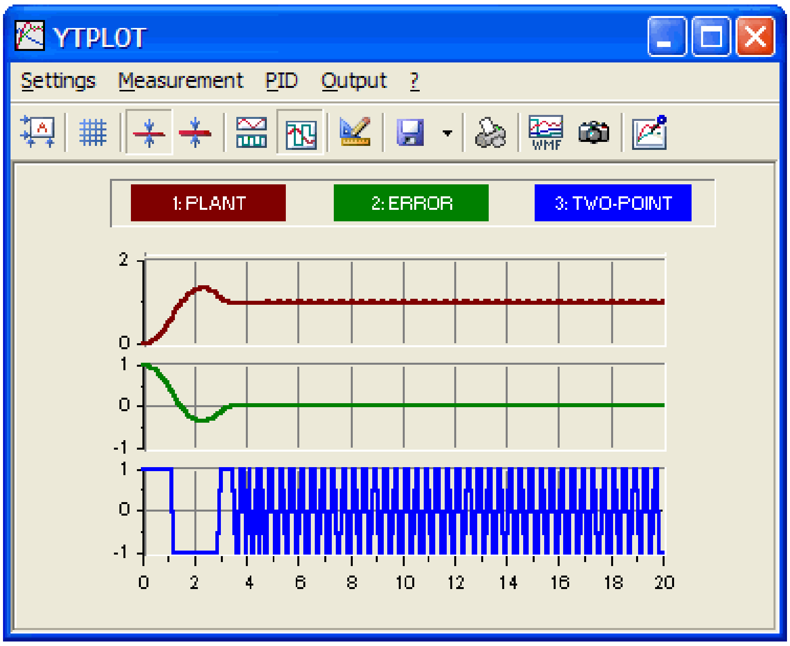


$T_V = 0$

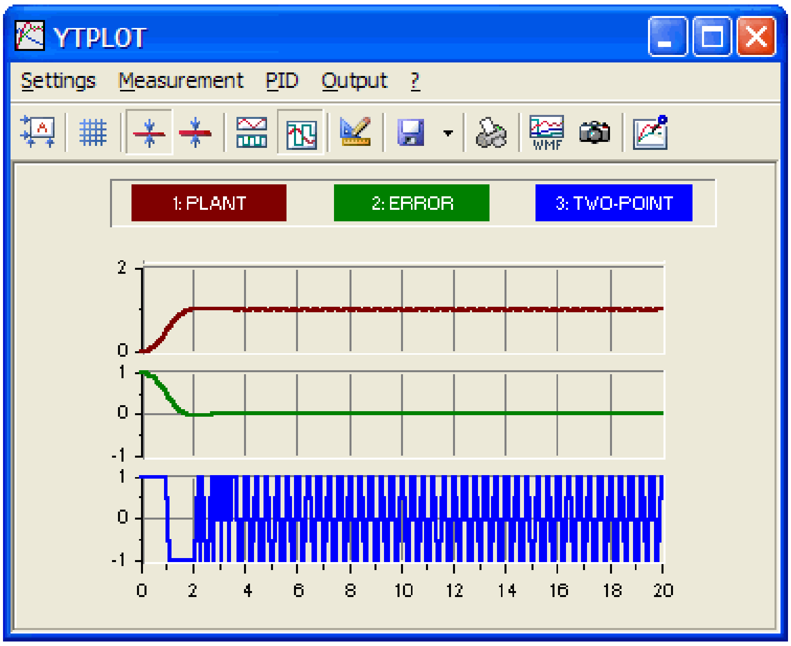
© Ingenieurbüro Dr. Kahlert 1991-2006


YTPLOT
Settings Measurement PID Output ?
1: PLANT
2: ERROR
3: TWO-POINT
2
0
1
0
-1
1
0
-1
0 2 4 6 8 10 12 14 16 18 20

$T_V = 0.1$


YTPLOT
Settings Measurement PID Output ?
1: PLANT
2: ERROR
3: TWO-POINT
2
0
1
0
-1
1
0
-1
0 2 4 6 8 10 12 14 16 18 20

$T_V = 0.3$



$T_V = 0.5$

For a value of $T_V = 0.3$ you get a usable compromise.

Related files:

SLIDMODE.BSY

Problem II.4: Controller design by the frequency response method

Problem specification: For a plant with the transfer function

$$G(s) = \frac{1}{(1+5s)(1+2s)(1+s)}$$

a PID-controller is to be designed which fulfills the following conditions:

- Gain crossover frequency $\omega_c > 0.6$
- Phase margin $\Phi_r > 50°$
- Open-loop gain $V > 40\text{dB}$ at $\omega < 0.002$

 © Ingenieurbüro Dr. Kahlert 1991-2006

Solution: The design can be done by using RESY. First we set the P-part of the controller to that value that the amplitude has at the gain crossover frequency (0.6) multiplied with -1. This is a value of 25dB resp. 17.78. Now the I-part is dimensioned in such a way that the current amplitude (including P-part) of the open-loop system has a value greater than 40 dB at 0.002, because the D-part will decrease this value later. We select a T_N of 17 (app. 53 dB at 0.002). Last but not least we have to adjust the required phase margin to ensure stability of the closed-loop system. The current phase of the open-loop (consisting of plant, P- and I-part) has a distance of -35° to -180° at the required gain crossover frequency. So we have to increase the phase by 35° + 50°. We select a T_V of 8.6. After doing this we see that the gain crossover frequency is not exactly at 0.6. So we set the P-part to 1 and re-calculate it as described above. After all these steps have been finished we get the following controller parameters:

$$K_R = 3.5 \quad T_N = 17 \quad T_V = 8.6$$

Related files:

FKL.UFK

Problem II.5: Root locus

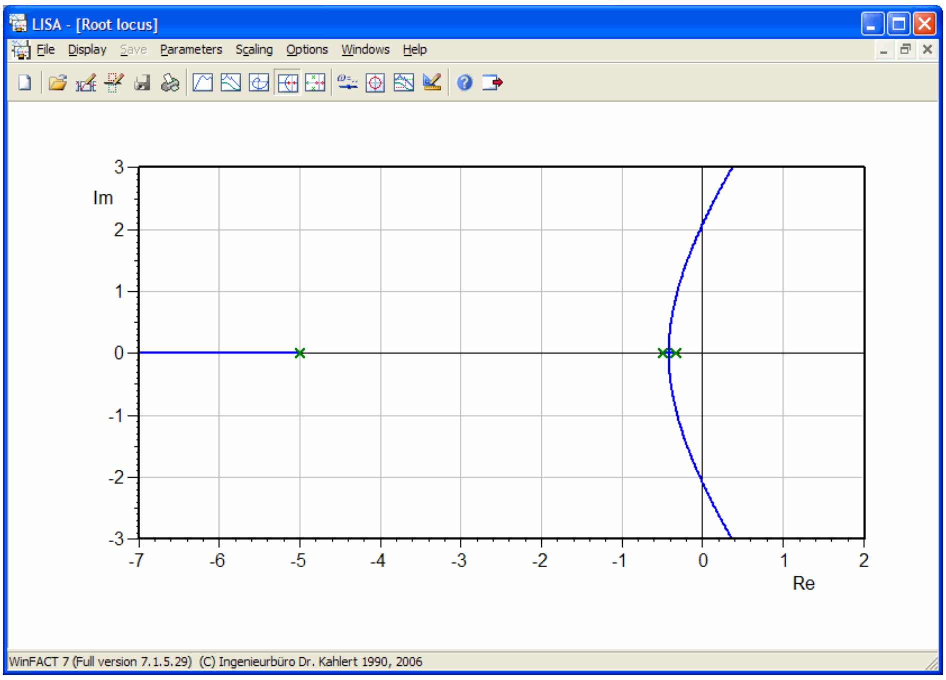
Problem specification: Given a plant with the transfer function

$$G(s) = \frac{1}{(1+0.2s)(1+2s)(1+3s)}.$$

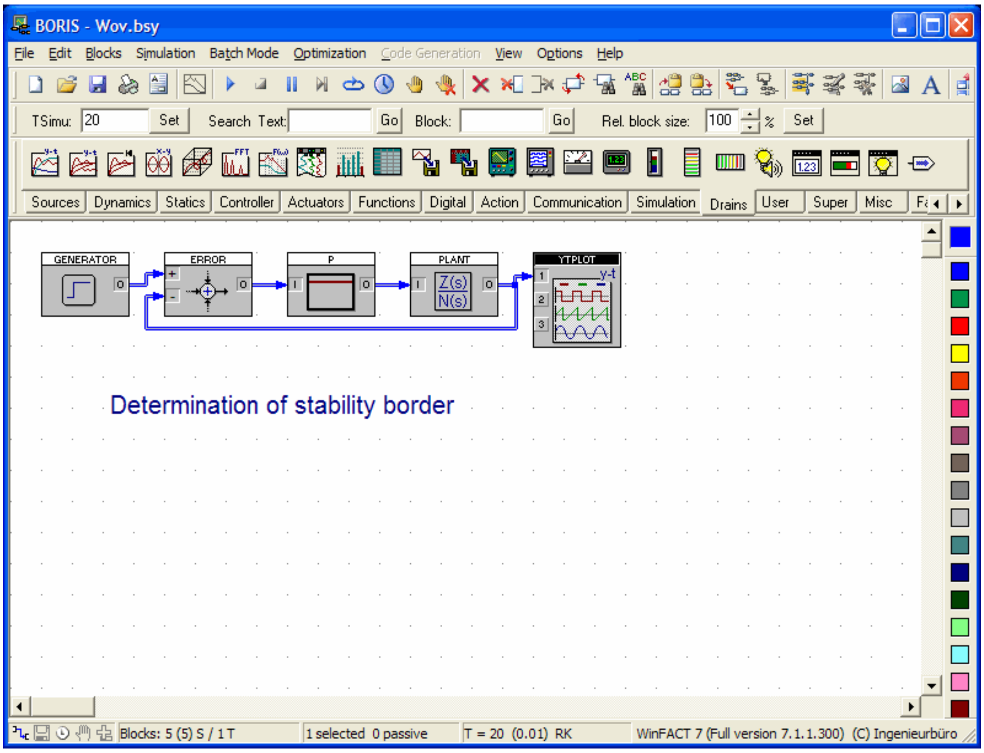
This plant is to be combined to a closed-loop system by inserting a P-controller.

Determine the corresponding root locus. Specifiy the controller gain K_R for which the closed-loop system becomes unstable.

Solution: The root locus can be determined by using LISA. We get the following result:



To determine the critical K_R value we can use the measurement mode of LISA or simulate the closed loop-system with BORIS; we want to do the latter. To get exact results we have to choose a very small simulation step size (e. g. $\Delta T = 0.01$) and the Runge-Kutta integration method. The screenshot below shows the corresponding simulation structure.



The simulation shows that the closed-loop system becomes unstable for $K_R \approx 29$. This value can be verified e. g. by using the Nyquist stability criterion with the Bode or Nyquist plot of the plant.

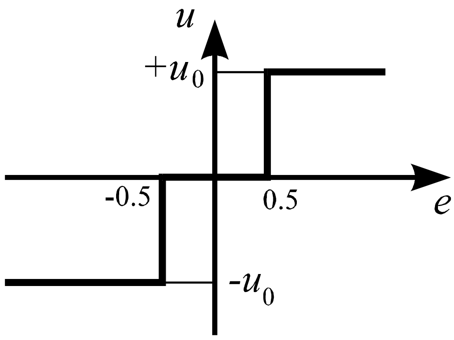
Related files: WOV.UFK
WOV.BSY

Problem II.6: Nonlinear control loop with undamped oscillation

Problem specification: Given a control loop with the plant

$$G(s) = \frac{1}{(1+0.5s)^3}$$

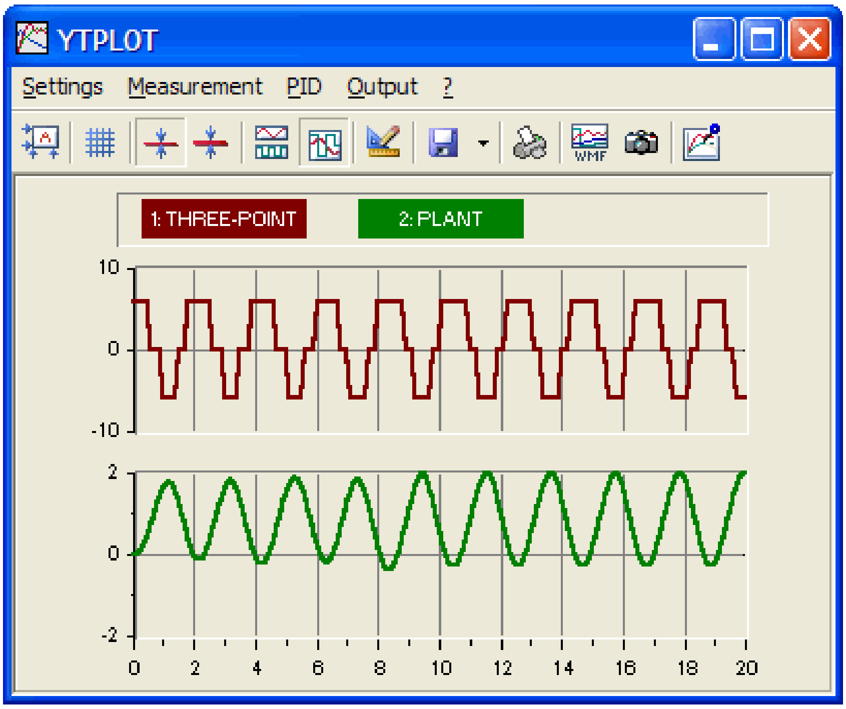
and the following three-point controller:



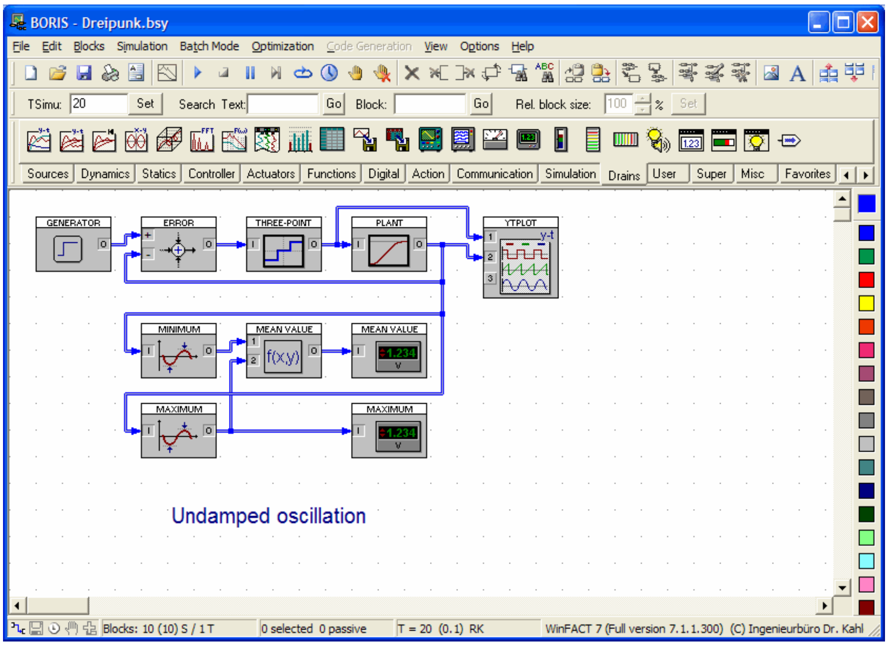
Determine the manipulated variable u_0 in such a way that the resulting oscillation does not exceed a maximum value of 2 for a step-sized reference signal and the mean value of the oscillation has a value of at least 0.8.

Solution: The amplitude of the oscillation increases with u_0. By determining the minimum and maximum of the oscillation an estimated mean value can be calculated which can be displayed via a digital meter. The manipulated variable u_0 is increased stepwise until the conditions are fulfilled. For $u_0 = 5.9$ we get the required results as shown in the screenshot below (upper curve: manipulated variable, lower curve: controlled variable).

© Ingenieurbüro Dr. Kahlert 1991-2006



The corresponding simulation system has the following structure:



Related files:

DREIPUNK.BSY

 © Ingenieurbüro Dr. Kahlert 1991-2006

Category III: Simulation

Problem III.1: Predator-prey system without capacity limitation [10]

Problem specification: The dynamics of a *prey population* x and a *predator population* y can be described by the differential equation system

$$\dot{x} = a\,x - b\,x\,y$$
$$\dot{y} = c\,x\,y - d\,y$$

It is

a: specific propagation rate of prey population

b: specific prey loss rate of prey population

c: specific prey profit rate of predator population

d: specific respiration rate of predator population

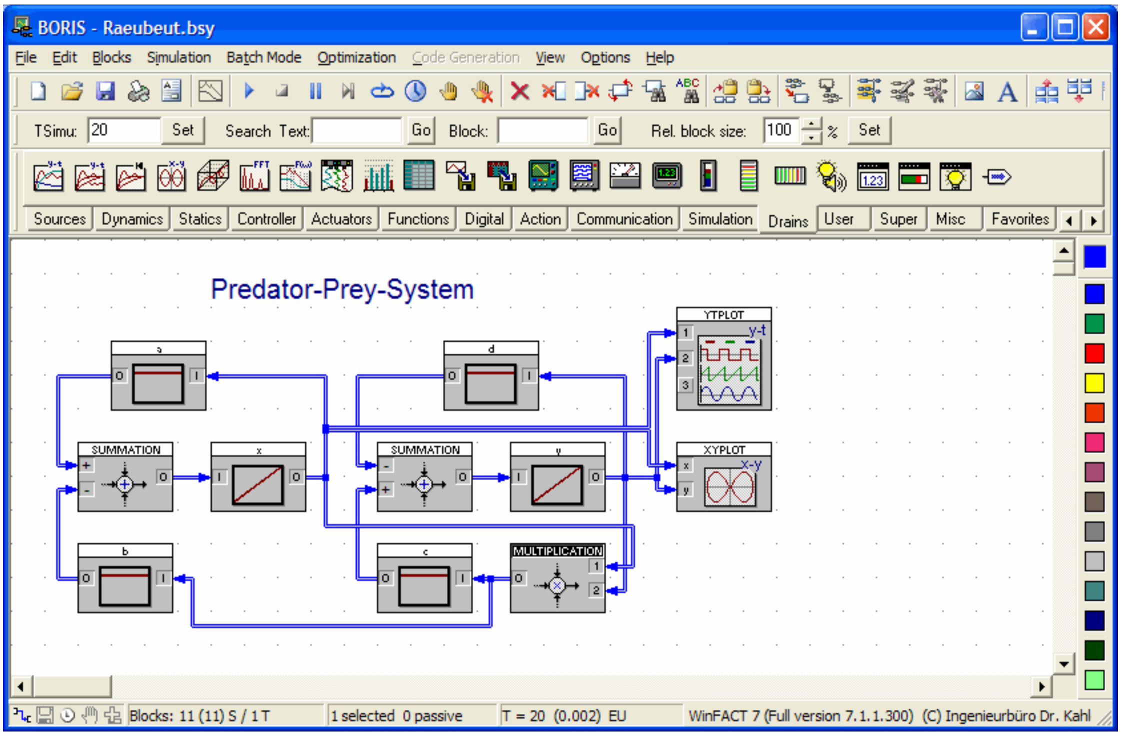
The interaction of predator and prey leads to loss in prey population and profit of predator population. If in spite of propagation of the prey the prey population decreases too much, this fact reduces the energy supply for the predator population and therefore its size. The "making prey" depends on the size of the prey population x as well as on the size of the predator population y. The proportional dependance from both variables leads to the nonlinearity xy. Corresponding loss is subtracted from the prey population (term bxy) and added to the predator population (term cxy). The loss of the prey population is partly compensated by the growth of the population (term ax). The predator needs the prey to compensate its normal respiration loss (parameter d) und thus to keep alive.

Simulate the given system by using BORIS for the following parameters:

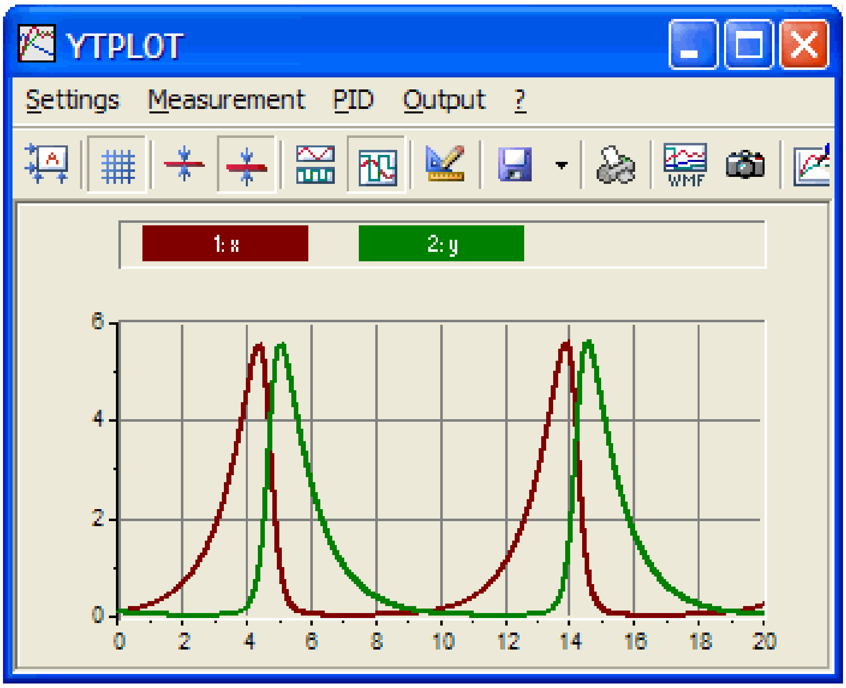
$$a = b = c = d = 1$$
$$x(t = 0) = y(t = 0) = 0.1$$
$$T_{\text{Simu}} = 20, \Delta T = 0.002$$

Determine the time responses $x(t), y(t)$ and the trajectory $y(x)$.

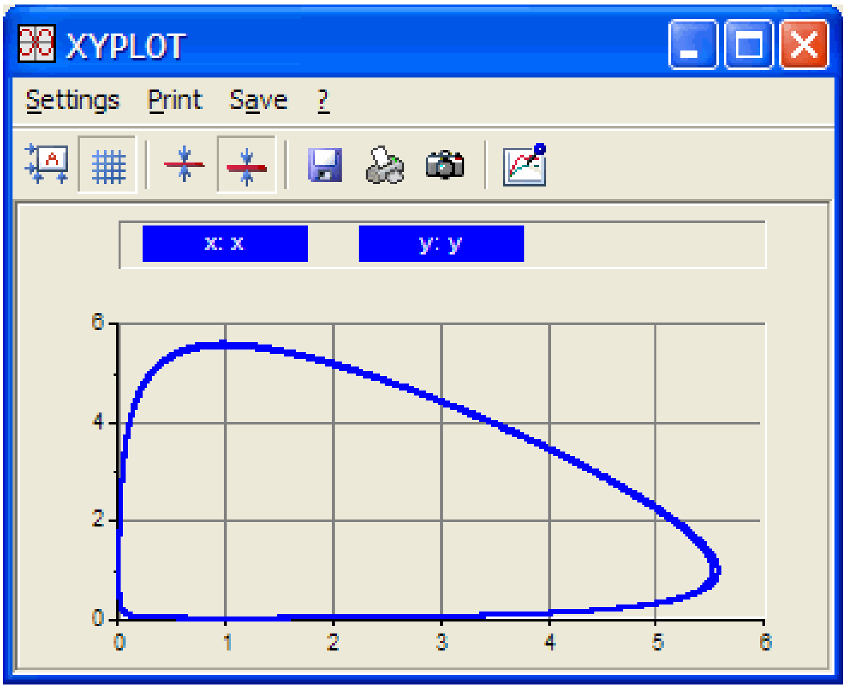
Solution: The simulation system has the following structure:



The simulation leads to the following results:



 © Ingenieurbüro Dr. Kahlert 1991-2006



Related files:

RAEUBEUT.BSY

Problem III.2: Tourism and environment [10]

Problem specification: A region that is especially attractive because of its natural environment is visited by a lot of tourists. The natural environment can regenerate up to a specific capacity border, but it is damaged and partly destroyed by tourism. Thus it loses attractivity and tourism decreases. This dynamic can be described by the differential equation system

$$\dot{x} = -a\,x + b\,y$$
$$\dot{y} = d\,y(1 - y/k) - c\,y$$

It is

x:	normalized number of tourists
y:	environment quality
a:	specific tourist loss rate
b:	advertisement effect
c:	specific environment destruction rate
d:	specific environment regeneration rate

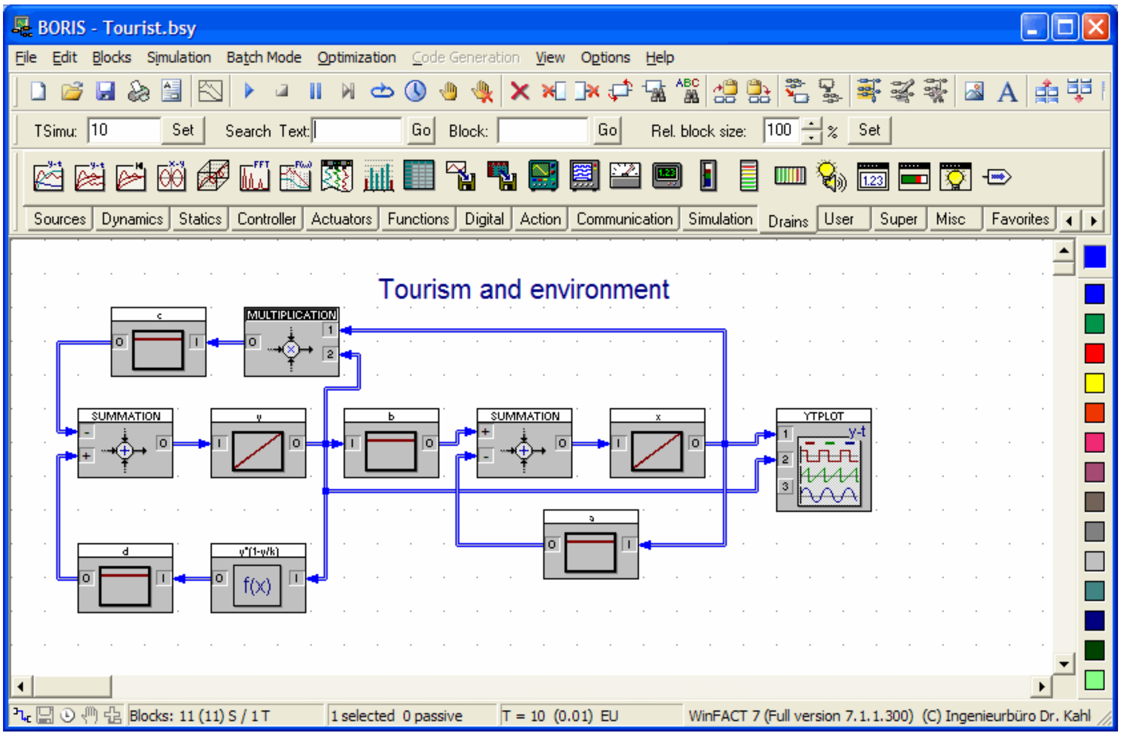
k: load capacity of environment

The attractivity of a region depends on its environment quality and can be intensified by advertisement (parameter *b*). The flow of tourists is proportional to this attractivity (term *by*) and increases the tourist population *y*. This population has permanent loss (parameter *a*) caused by tourists going home. The environment influence caused by the tourists depends on the number of tourists and the environment quality itself and thus is proportional to *xy* and a specific destruction rate *c*. Left alone after an initial destruction the environment would regenerate again to the capacity border *k* with a specific regeneration rate *d*.

Determine the time responses $x(t)$, $y(t)$ by using BORIS for the following parameters:

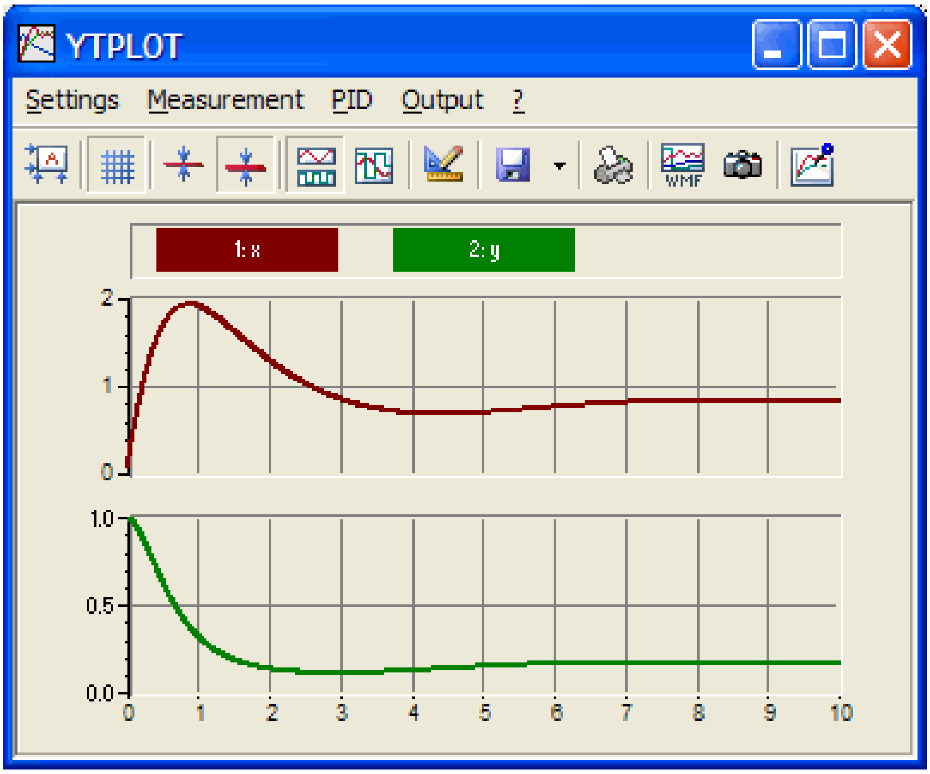
$$a = c = d = k = 1 \qquad b = 5$$
$$x(t = 0) = 0.1 \qquad y(t = 0) = 1$$
$$T_{\text{Simu}} = 10, \;\; \Delta T = 0.01$$

Solution: The simulation system has the following structure:



We get the following results:

© Ingenieurbüro Dr. Kahlert 1991-2006



Related files:

TOURIST.BSY

Problem III.3: Rotation pendulum [10]

Problem specification: A massless stiff stick of length r can rotate in a vertical plane. At the end of the stick a mass m is located. If the pendulum is pushed with a large angular velocity y it will rotate mutliple times before it begins to oscillate. This oscillation is damped by the damping d; thus the pendulum will stop after some time at its lower dead point. The dynamic of the pendulum is given by the differential equation system

$$\dot{x} = y$$

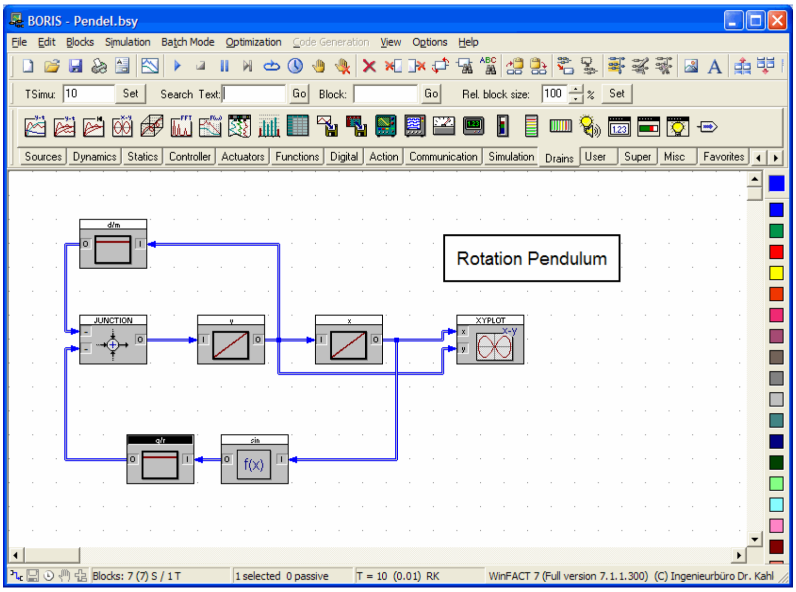
$$\dot{y} = -\frac{g}{r}\sin x - \frac{d}{m} y \qquad g = 9.81\,\mathrm{m/s^2}$$

x is the angle of deflection of the pendulum.

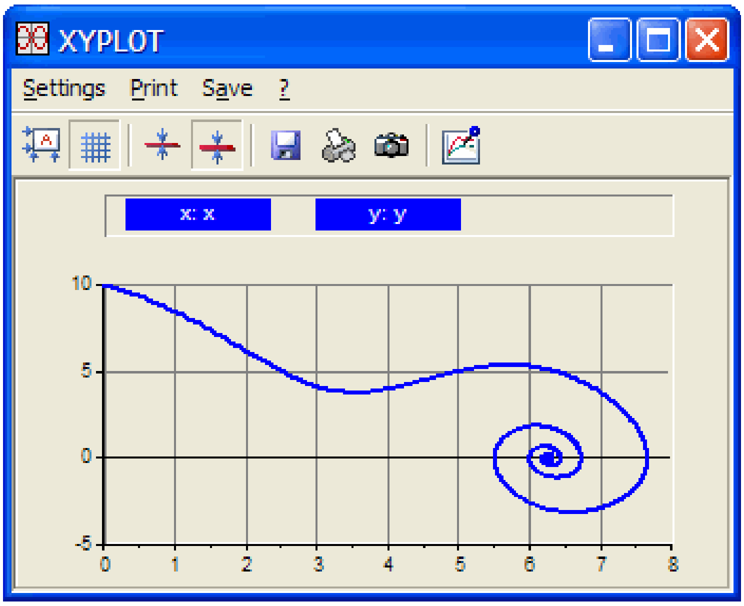
Determine the course of the trajectory $y(x)$ with BORIS for the following parameters:

$$m = r = d = 1$$
$$x(t = 0) = 0$$
$$y(t = 0) = 10$$
$$T_{\text{Simu}} = 10, \quad \Delta T = 0.01$$

Solution: The system has the following simulation structure:



We get the following results:



Related files:

PENDEL.BSY

Problem III.4: Chaotic bistable oscillator [10]

Problem specification: This system consists of a linear oscillator with a negative feedback of x^3 to y. The system is stimulated sinusiodal what results in a chaotic behaviour. The corresponding differential equation system is given by

$$\dot{x} = y$$

$$\dot{y} = x - x^3 - d\,x + q\cos\omega t$$

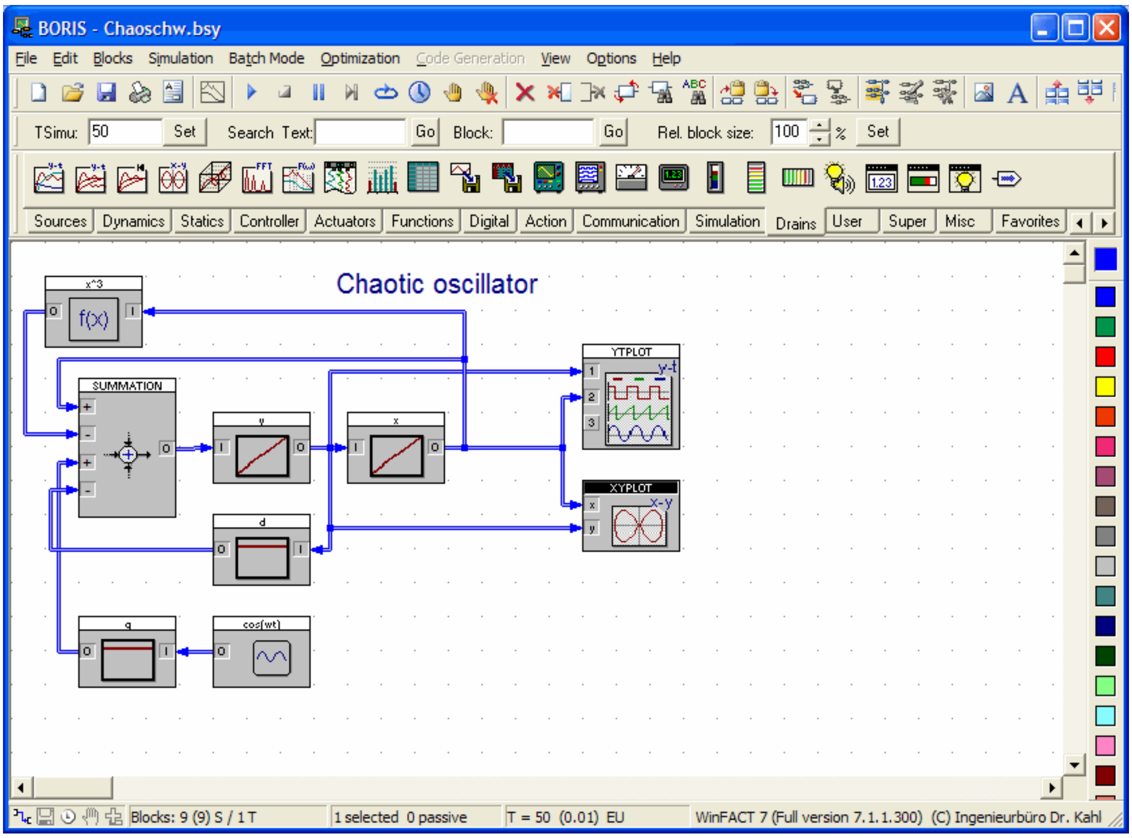
Determine the time responses $x(t)$, $y(t)$ and the trajectory for the following parameters:

$$d = 0.25 \quad q = 0.3 \quad \omega = 1$$

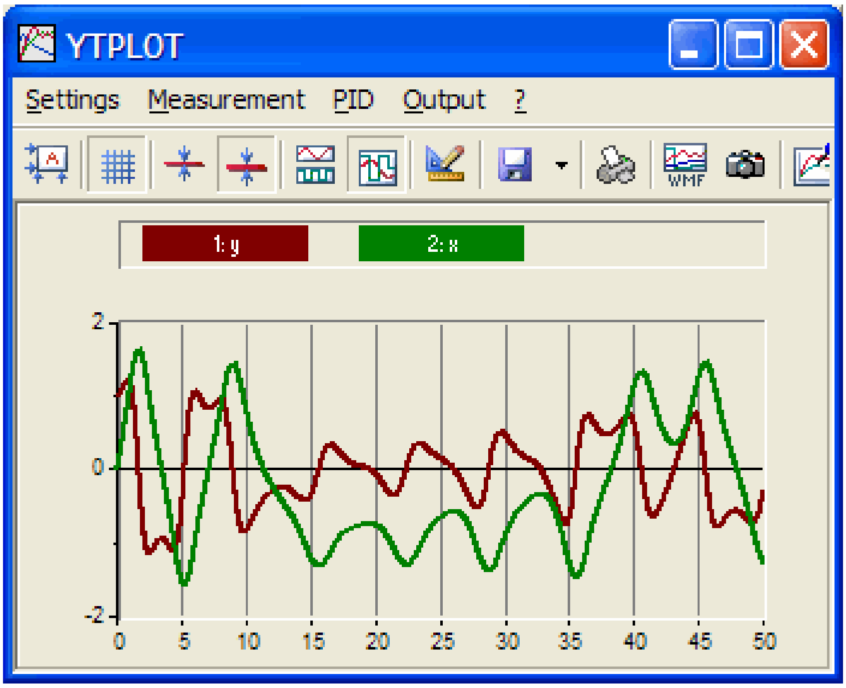
$$x(t=0) = 0 \quad y(t=0) = 1$$

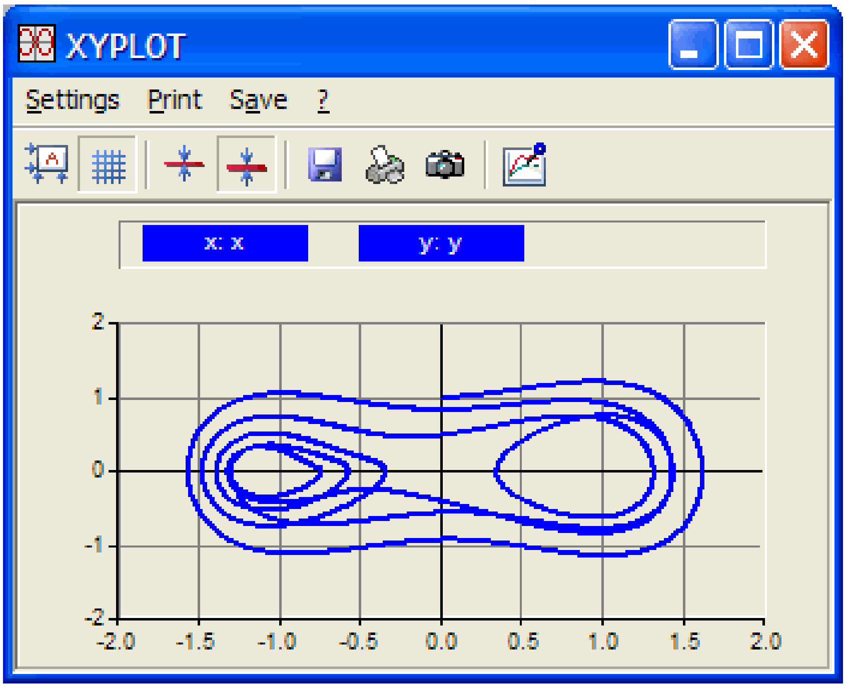
$$T_{\text{Simu}} = 50, \ \ \Delta T = 0.01$$

Solution: The system has the following simulation structure:



We get the following simulation results:





Related files:

CHAOSCHW.BSY

© Ingenieurbüro Dr. Kahlert 1991-2006

Problem III.5: Lorenz system [10]

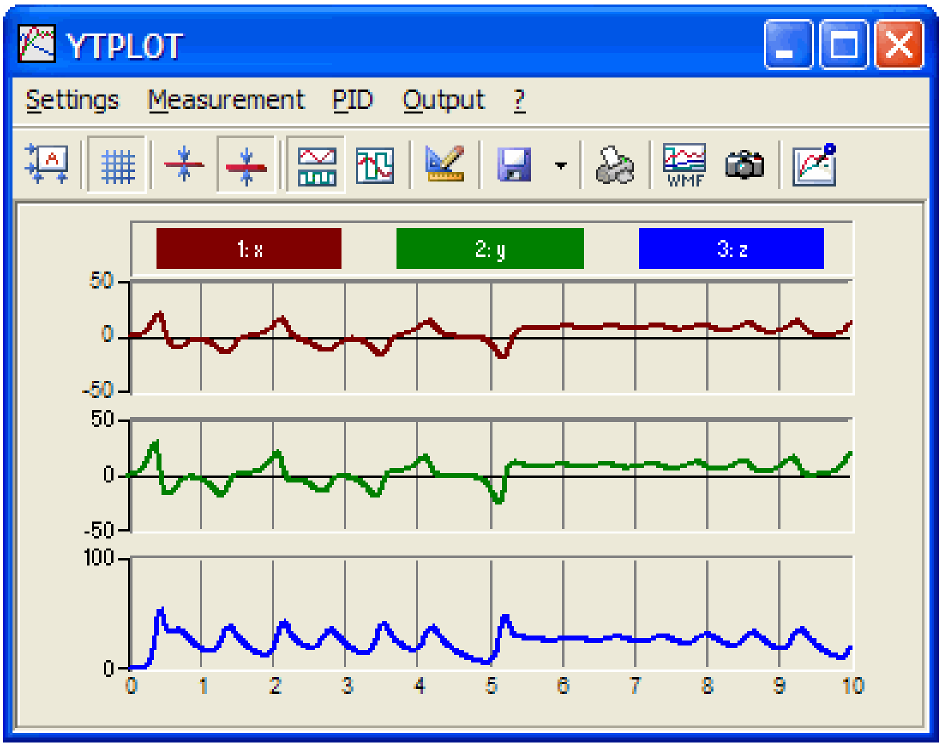
Problem specification: The Lorenz system is an approximated representation of the hydro-thermodynamical equations for coupling of heat convection and heat conduction in fluid current. The state variable x describes the velocity profile, the state variables y and z the temperature distribution. If the system parameters are within a specific range the system shows chaotic behaviour. The differential equation system is given by

$$\dot{x} = a(y - x)$$
$$\dot{y} = -xz + bx - y$$
$$\dot{z} = xy - cz$$

Determine the time responses $x(t), y(t), z(t)$ by using BORIS for

$$a = 10, b = 28, c = 2.667$$
$$x(t = 0) = 1, \ \ y(t = 0) = z(t = 0) = 0$$
$$T_{\mathrm{Simu}} = 10, \Delta T = 0.01$$

Solution: We get the following simulation results:



Related files:

LORENZ.BSY

Problem III.6: Coupled dynamos [10]

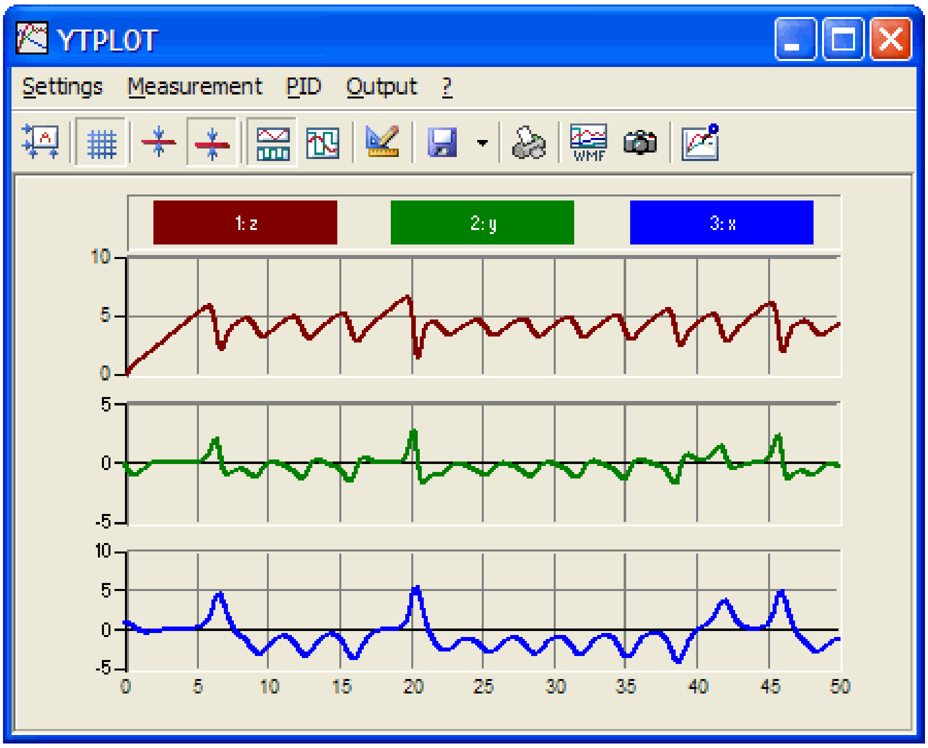
Problem specification: Two identical dynamos are coupled; the current of the first dynamo stimulates the magnetic field of the second one and vice versa. The currents are the state variables x and y. State variable z is the rotation speed of the first dynamo. The parameter c specifies the difference of the rotation speeds of both dynamos. The system shows chaotic behaviour which is given by the differential equation system

$$\dot{x} = z\,y - a\,x$$
$$\dot{y} = (z - c)x - a\,y \qquad c = a(b^2 - 1/b^2)$$
$$\dot{z} = 1 - x\,y$$

Determine the response of all state variables for the following parameters:

$$a = 1, \quad b = 2$$
$$x(t = 0) = 1$$
$$y(t = 0) = z(t = 0) = 0$$
$$T_{\mathrm{Simu}} = 50, \quad \Delta T = 0.01$$

Solution: We get the following simulation results:



 © Ingenieurbüro Dr. Kahlert 1991-2006

Related files:

DYNAMOS.BSY

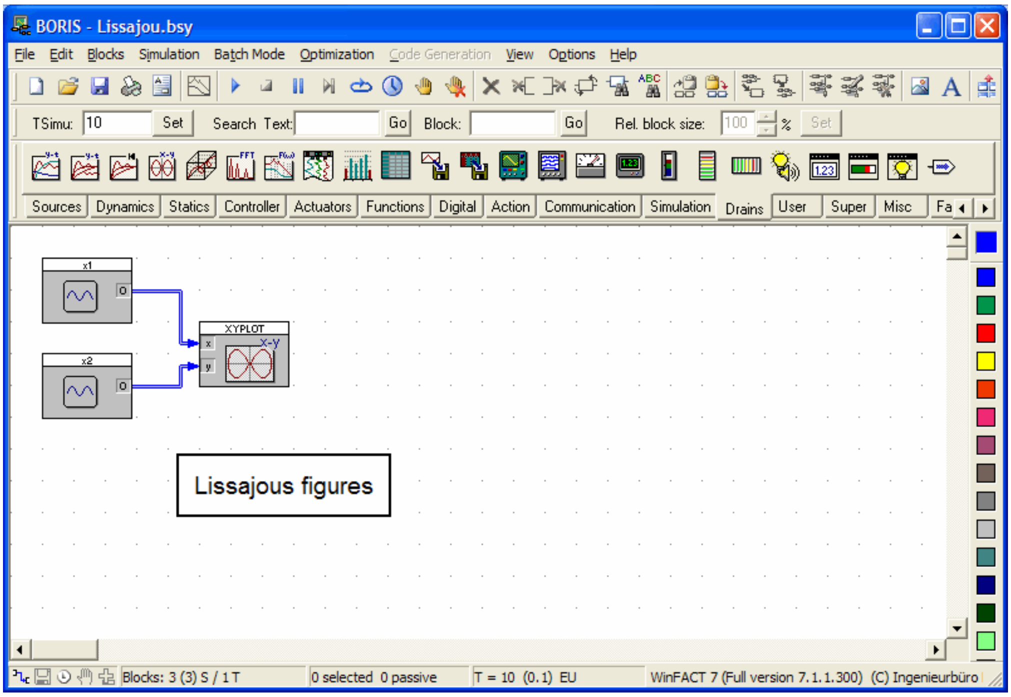
Problem III.7: Lissajous figures

Problem specification: Given the two time signals

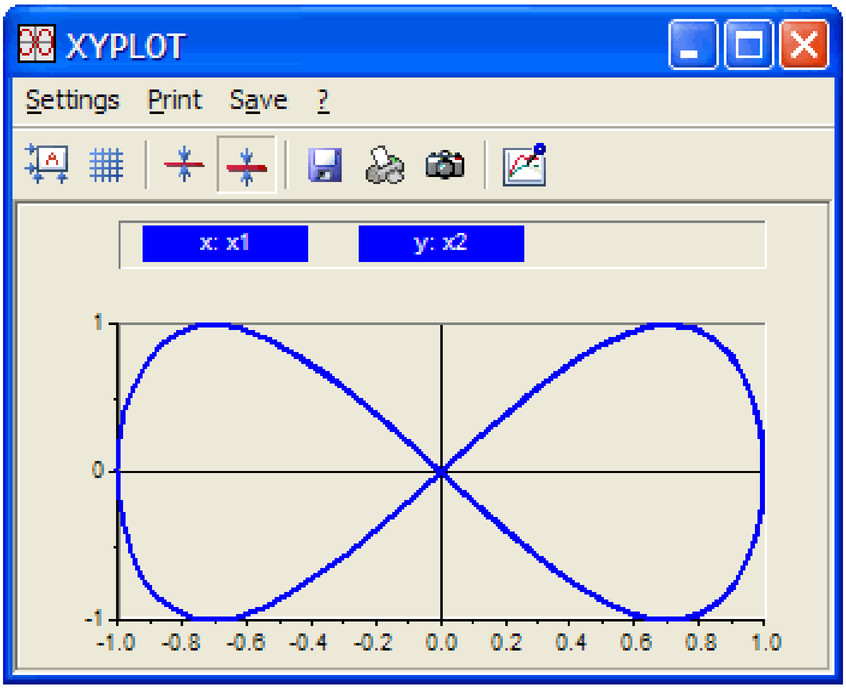
$$x_1(t) = \sin t$$
$$x_2(t) = \sin n\,t$$

Determine the trajectories $x_2(x_1)$ for $n = 2$, 4 and 6 and a simulation length of 10.

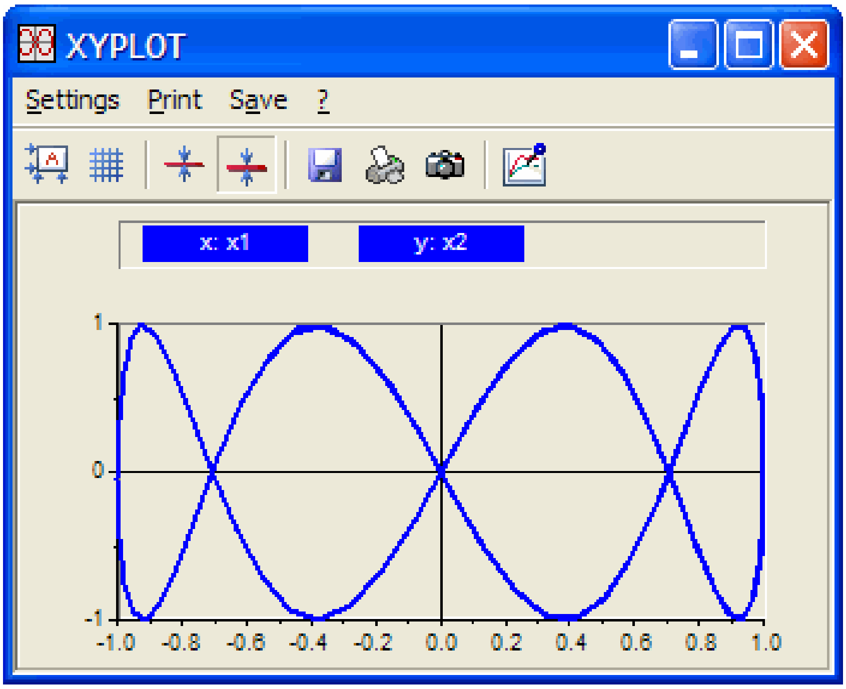
Solution: We use two signal generators of BORIS in the sinus generator mode and a x-y-plot to display the results. The screenshot below shows the simulation structure.



Depending on the choice of n we get the well known Lissajous figures:

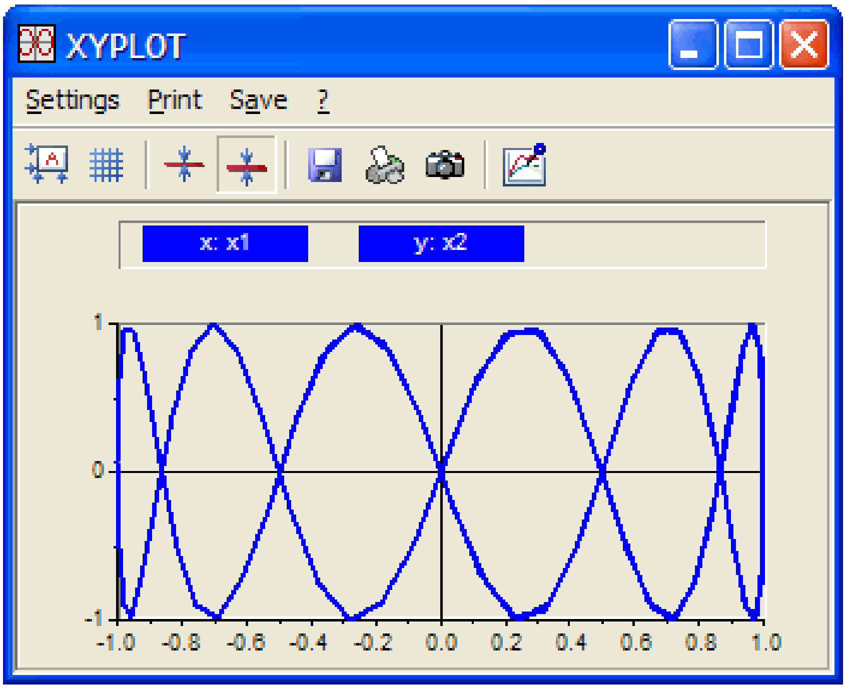


n = 2



n = 4

© Ingenieurbüro Dr. Kahlert 1991-2006



n = 6

Related files:

LISSAJOU.BSY

Problem III.8: Van der Pol differential equation [12]

Problem specification: Given the Van der Pol differential equation

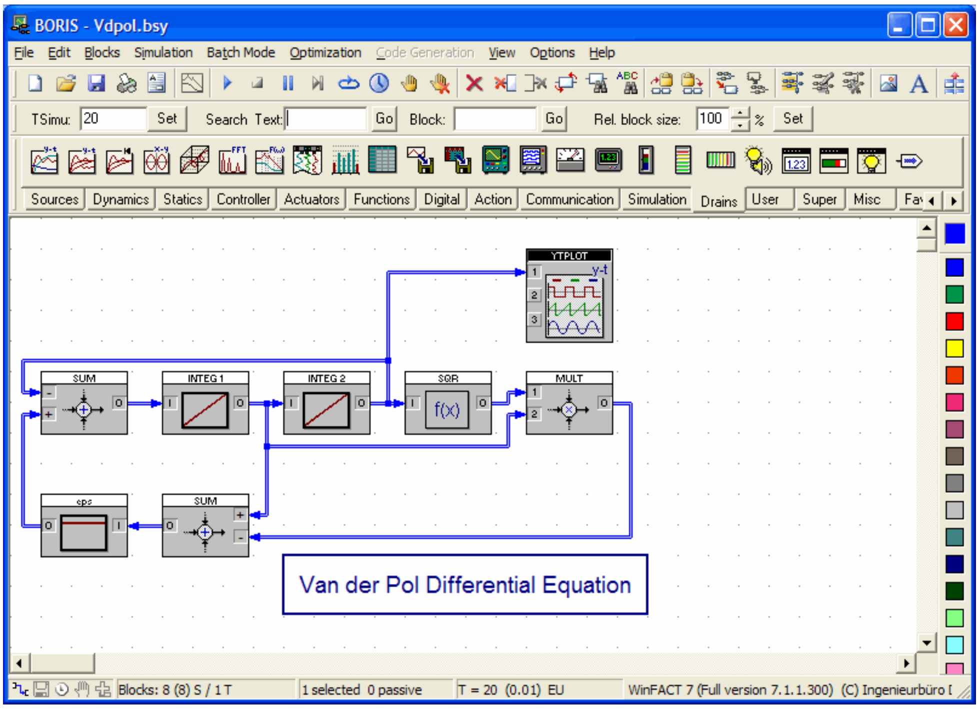
$$\ddot{y} + \varepsilon(y^2 - 1)\dot{y} + y = 0$$

Use BORIS to calculate the solution of this equation for the following parameters:

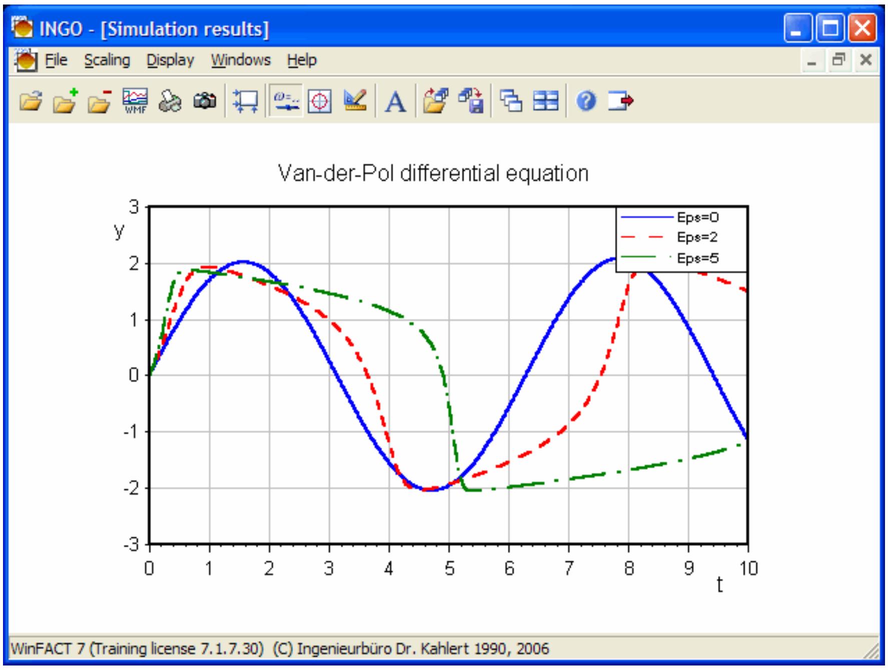
$$y(t=0) = 2 \quad \dot{y}(t=0) = 0$$
$$\varepsilon = 0,\ \ 2,\ \ 5$$
$$T_{\text{Simu}} = 20,\ \ \Delta T = 0.01$$

Solution: The system is of second order and has the following simulation structure:

© Ingenieurbüro Dr. Kahlert 1991-2006



The simulation results for different values of ε can be saved in a file and be compared later with INGO. We get the following results:



The value of the parameter ε is responsible for the nonlinear behaviour of the system. For the special case ε = 0 we get a linear oscillator.

Related files:

VDPOL.BSY

 © Ingenieurbüro Dr. Kahlert 1991-2006

Problem III.9: Fast Fourier Transformation

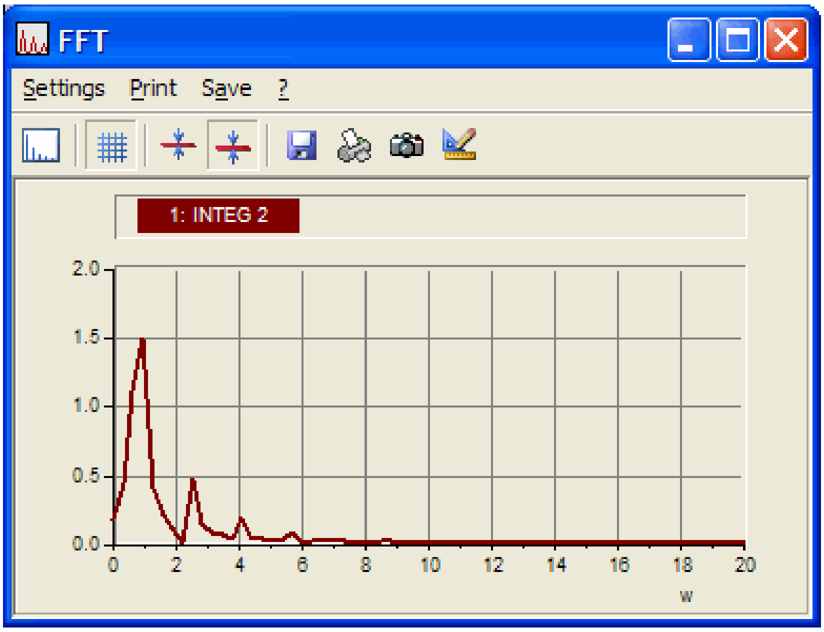
Problem specification: Given the Van der Pol differential equation

$$\ddot{y} + \varepsilon(y^2 - 1)\dot{y} + y = 0$$

Use BORIS to calculate the amplitude spectrum of $y(t)$ for the following parameters:

$$y(t=0) = 2 \quad \dot{y}(t=0) = 0$$
$$\varepsilon = 0,\ 2,\ 5$$
$$T_{\text{Simu}} = 20,\ \Delta T = 0.01$$

Solution: The structure known from the previous Problem is extended by a FFT block. We get the following result:



Related files:

FFT.BSY

Problem III.10: Stability of integration methods [12]

Problem specification: Given a PT1-element with the transfer function

$$G(s) = \frac{K}{1+Ts}$$

and the parameters

$$K = 1,\ T = 2$$

Simulate the behaviour of the free system, i. e. for the case $u(t) \equiv 0$, and an initial state of $y(0) = 1$ for different simulation step sizes ΔT

- using the Euler method

- using the Runge-Kutta method

Compare the results with the exact solution

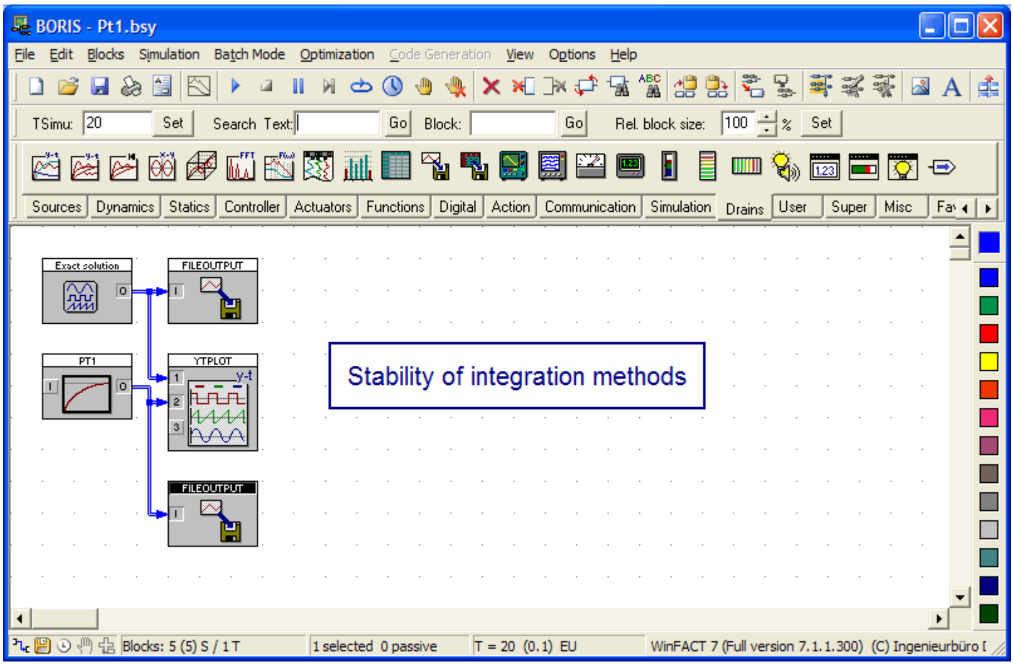
$$y(t) = e^{-t/2}.$$

For which simulation step size do the integration methods become unstable?

Solution: This Problem shall elucidate the influence of the simulation step size on the exactness of the simulation result and how to find a proper step size in dependance of the dynamic of the simulated system. Normally in case of linear systems the step size should be less than 1/10 of the smallest time constant of the system - i. e. less than 1/10 of 2 in this sample.

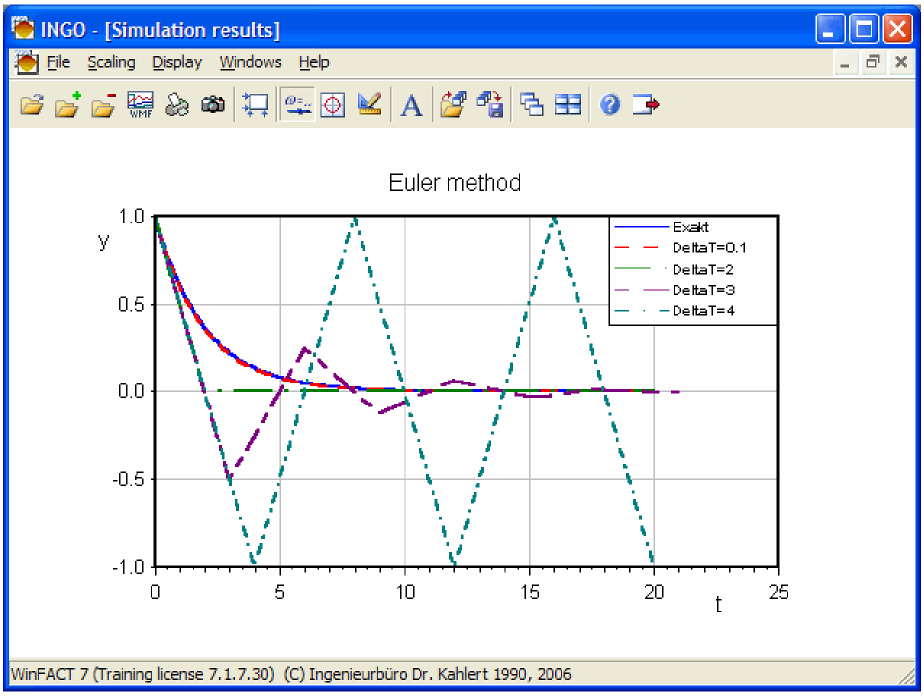
The simulation can be executed with BORIS; the input of the PT1-block can remain open. The results can be saved and compared with INGO.

The screenshot below shows the simulation structure:



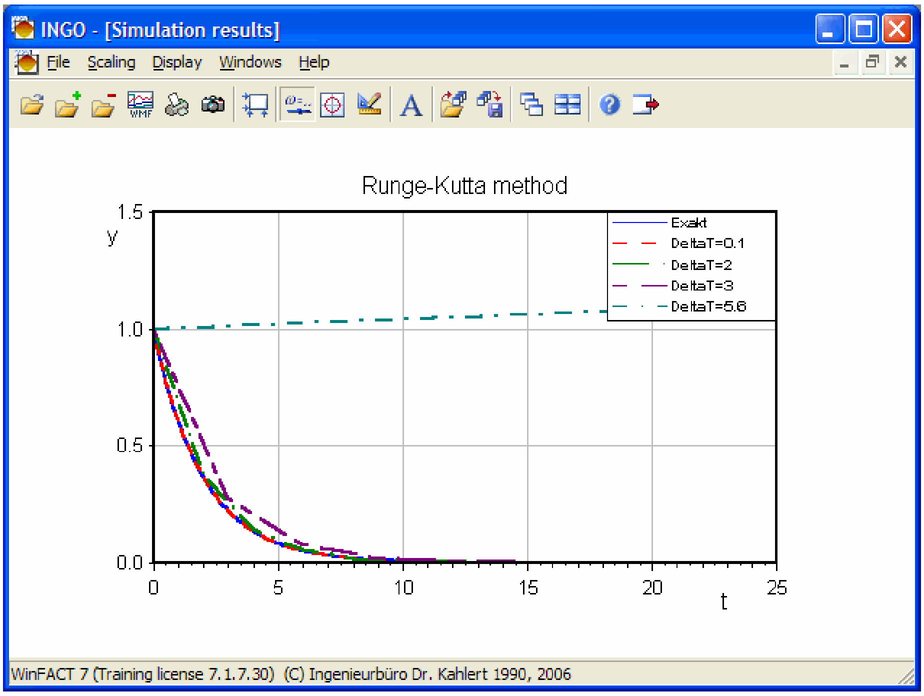
 © Ingenieurbüro Dr. Kahlert 1991-2006

The exact solution can be calculated by using a generator which is programmed via the function parser. The screenshot below shows the simulation results calculated by using the Euler integration method for step sizes of $\Delta T = 0.1,\ 2,\ 3$ and 4.



We recognize that for a step size of 0.1 the simulation result is nearly identical to the exact solution. This step size is 1/20 of the system time constant and thus small enough. If the step size is increased the simulation result becomes more and more inexact; for $\Delta T = 4$ the Euler method becomes unstable.

The results for the Runge-Kutta method are shown in the screenshot below. We can recongnize that this method becomes unstable not before a step size of approximately 5.6.



Related files:

PT1.BSY

Problem III.11: Stiff systems

Problem specification: Given a PT2-element with the transfer function

$$G(s) = \frac{1}{(1+s)(1+100s)}.$$

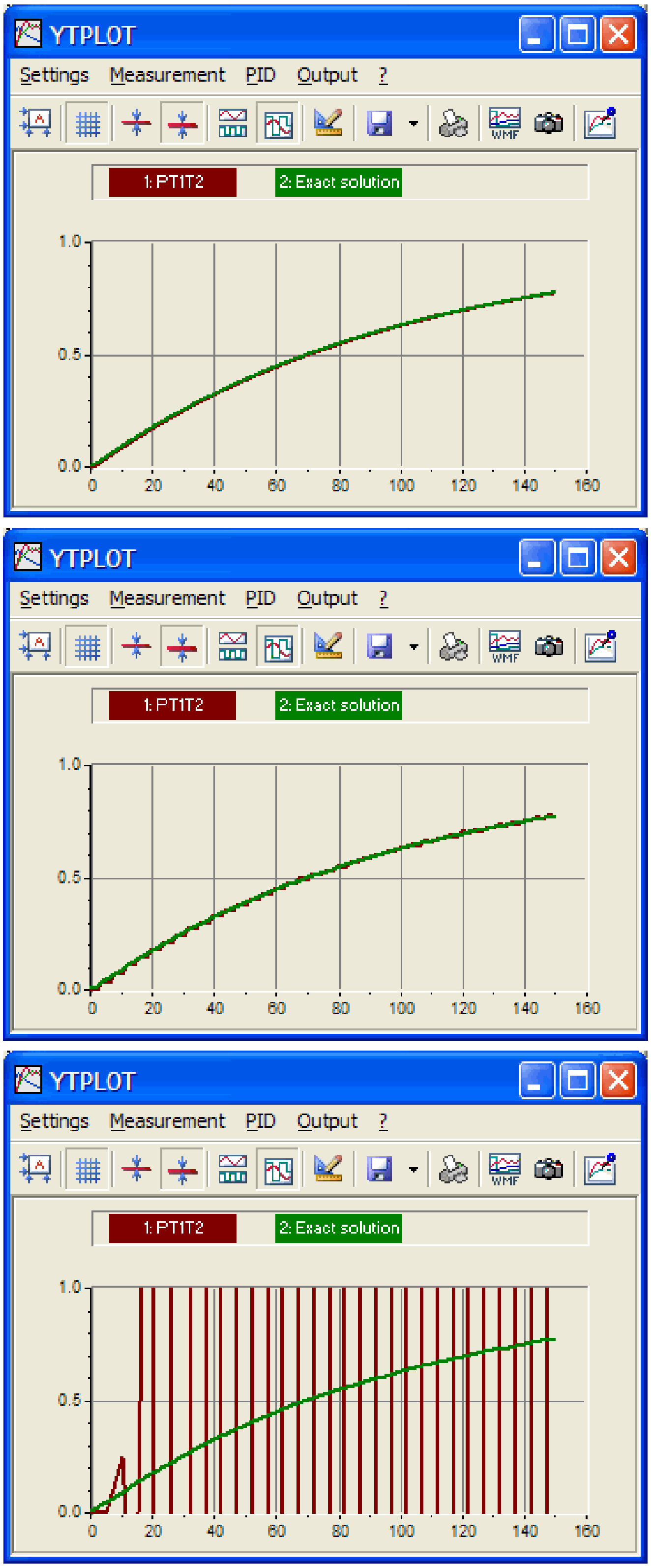
Determine the step response of the system by using the Euler integration method for simulation step sizes of ΔT=0.1, 2, 5 up to a simulation time of 150 and compare the results with the exact solution given by

$$y(t) = 1 + \frac{1}{99}e^{-t} - \frac{100}{99}e^{-t/100}.$$

Solution: The given system is a so-called "stiff system" with the time constants $T_1 = 1$ and $T_2 = 100$. If a very small step size is chosen you get exact results but the simulation needs much time. If a greater step size is chosen the results become inexact resp. the simulation becomes unstable.

We get the following results:

 © Ingenieurbüro Dr. Kahlert 1991-2006



Results for $\Delta T = 0.1$ *(top),* $\Delta T = 2$ *(middle),* $\Delta T = 5$ *(bottom)*

Related files:

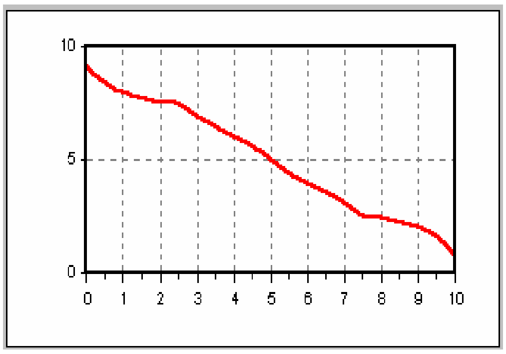
STEIFSYS.BSY

Category IV: Fuzzy logic

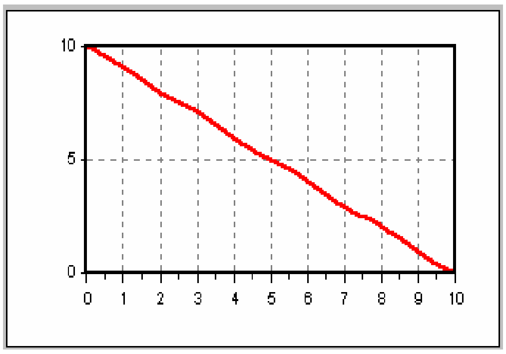
Problem IV.1: Characteristic curve generation

Problem specification: Define one input and one output variable with five fuzzy sets each. Distribute these sets uniformly over the range from 0 to 10 by choosing the standard distribution. Try to create the following characteristic curves:

a)

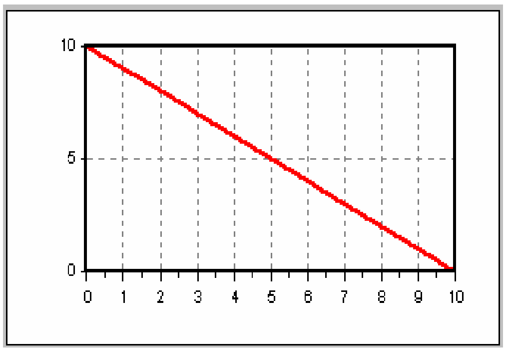


b)

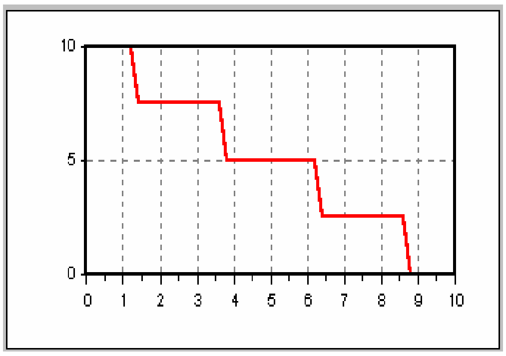


© Ingenieurbüro Dr. Kahlert 1991-2006

c)



d)



Solution:

a) Center of gravity method (Max-Prod-inference)

The characteristic curve shown in a) does not reach the borders (0, 10) and (10, 0). The curve has light inflections at the ends while it's smooth in the middle. The inflections result from the unsymmetrical sets at the range borders.

b) Modified center of gravity method (Max-Min-inference)

Characteristic curve b) reaches the borders. The inflections of curve a) are not existent in this case because the border sets are extended symmetrically.

c) Approximated center of gravity method

Because all output sets are viewed as singletons and these are distributed uniformly over the range of the linguistic variable the characteristic curve is a straight line.

d) Maximum height (right/left)

The characteristic curve changes stepwise because only the output set for the rule with the greatest match of degree is used for defuzzification.

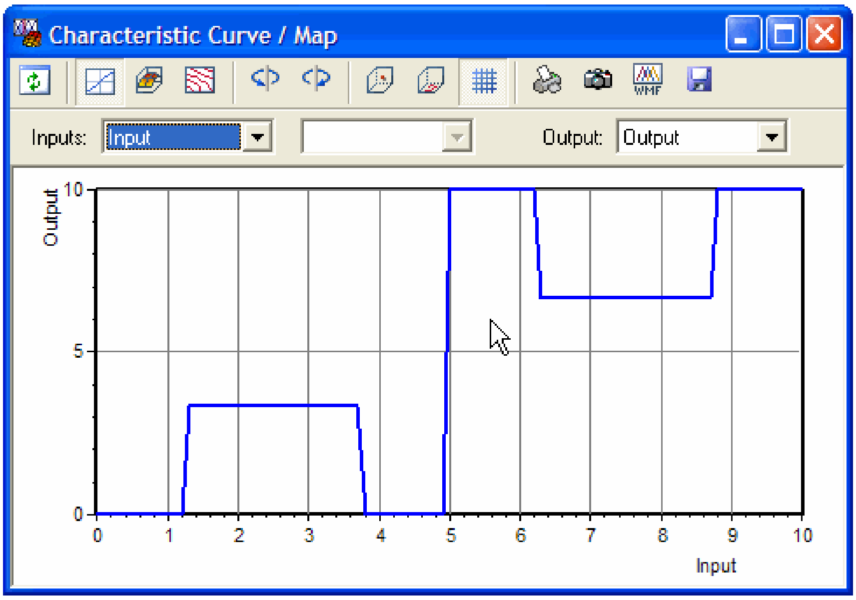
© Ingenieurbüro Dr. Kahlert 1991-2006

Related files:

DEFUZZY.FUZ

Problem IV.2: Fuzzy logic with one input and one output

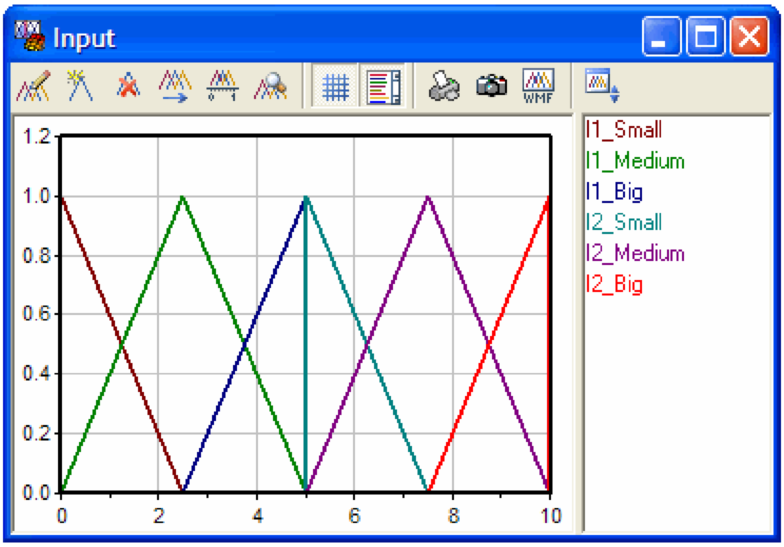
Problem specification: The following problem is to be solved: You want to use a microcontroller board for fuzzy logic but only one input is still available. Use this input in such a way that it is possible to handle two different signals separately. For the linguistic input variable define six proper fuzzy sets, for the output variable define four sets in standard form. Consider how the input sets, the rule base and the defuzzification have to be selected to generate the following characteristic curve:



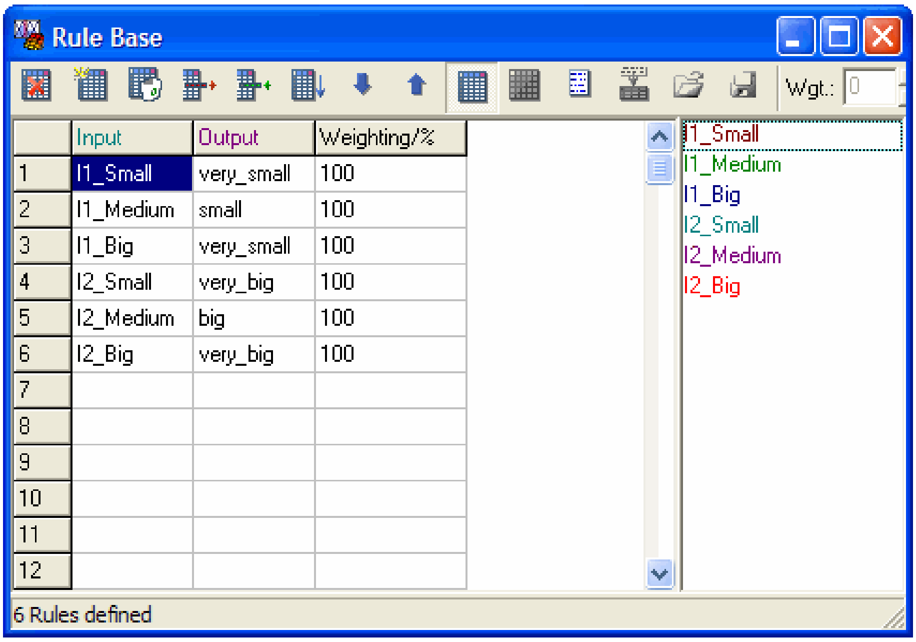
How have both input signals to be processed before they are sent into the fuzzy module?

Solution: The input fuzzy sets have to be specified as follows:

© Ingenieurbüro Dr. Kahlert 1991-2006

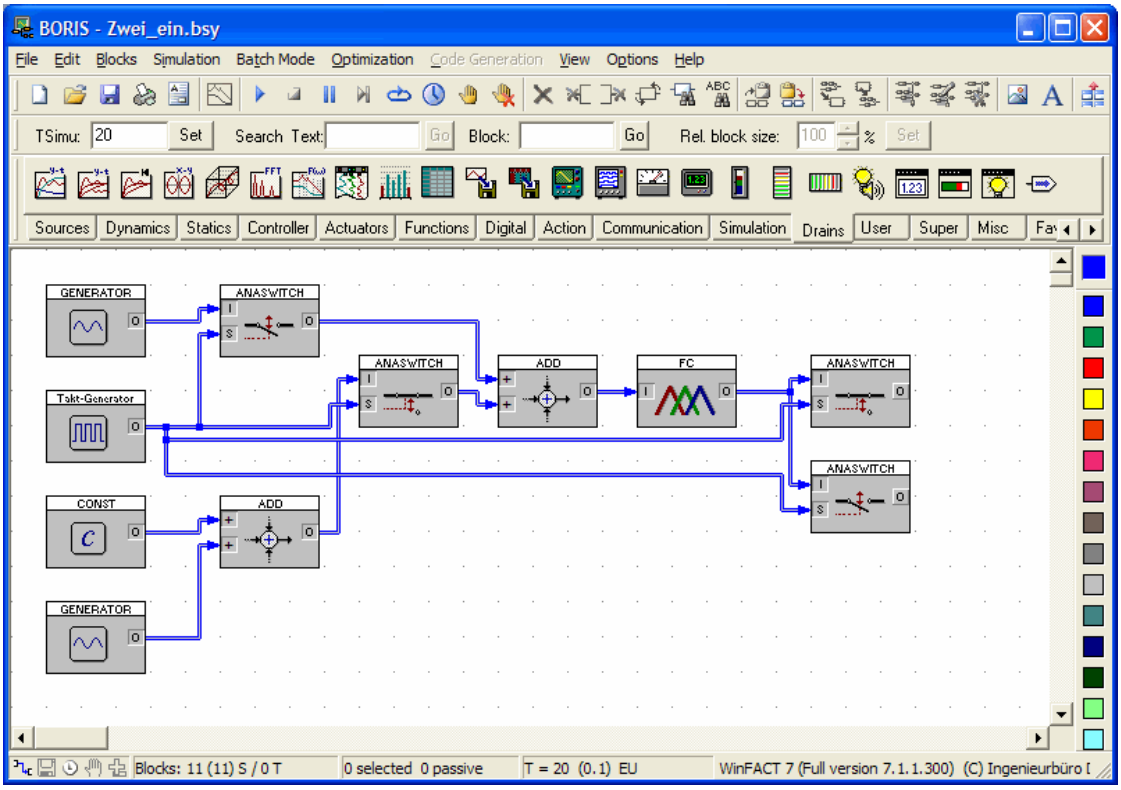


It can be seen clearly that in the middle of the range two sets meet together. If only the lower resp. upper half of the range is considered, a standard configuration can be recognized. The rule base is as follows:



Because the characteristic curve has only horizontal segments the defuzzification must be executed via the maximum height method (see Problem IV.1 d).

A little bit more complicated is the signal pre-processing. Corresponding to the definition of the linguistic input variable both input signals should use different amplitude ranges: The first signal the range from 0 to 5, the second signal the range from 5 to 10. In the following both signals are sampled with a sufficiently high frequency. Thus the input signal processing might look as follows:

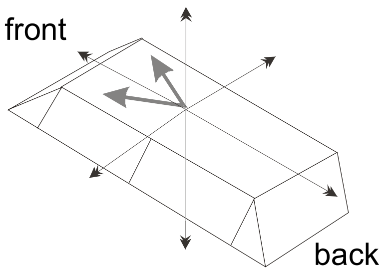


The output signals of both relays can be processed in an appropriate way later.

Related files: ZWEI_EIN.FUZ
ZWEI_EIN.BSY

Problem IV.3: Fuzzy logic with two inputs

Problem specification: A remote controlled solar toy car shall drive to a light source automatically to reload its accumulators if these get empty. The car has two light sensors mounted on its roof as illustrated in the figure below (gray arrows).

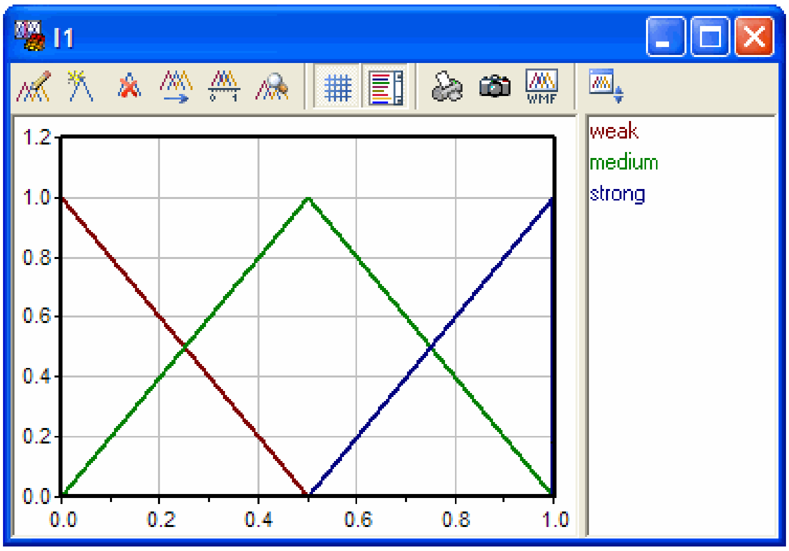


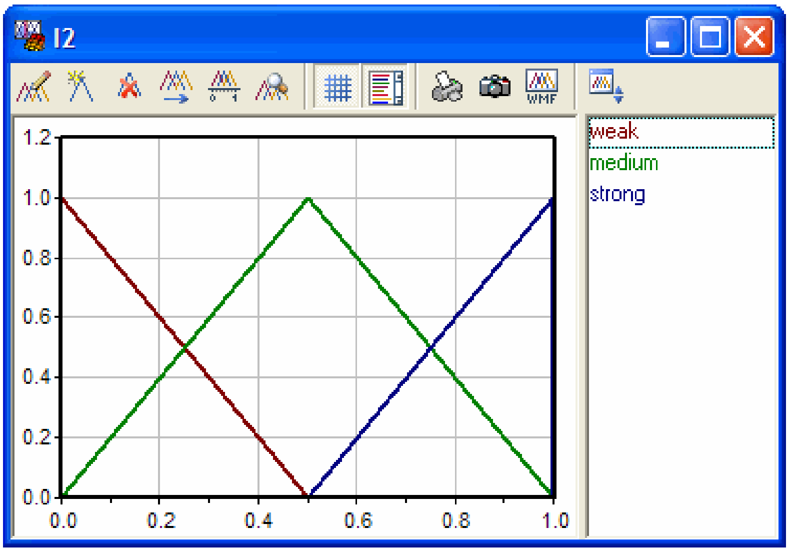
Design a fuzzy logic for the light search. For both inputs (light sensors) select three fuzzy sets between 0 and 1, for the output (driving direction) five sets

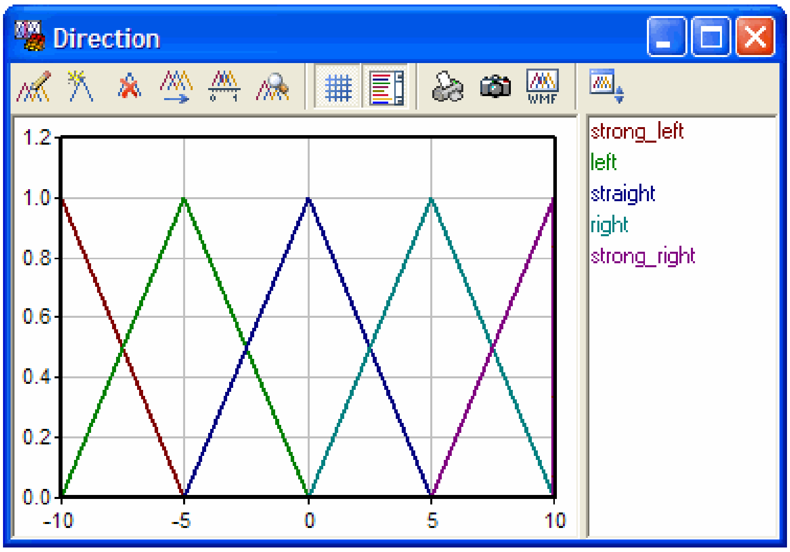
© Ingenieurbüro Dr. Kahlert 1991-2006

between -10 and 10. Select the defuzzification in such a way that the complete range of the output is used.

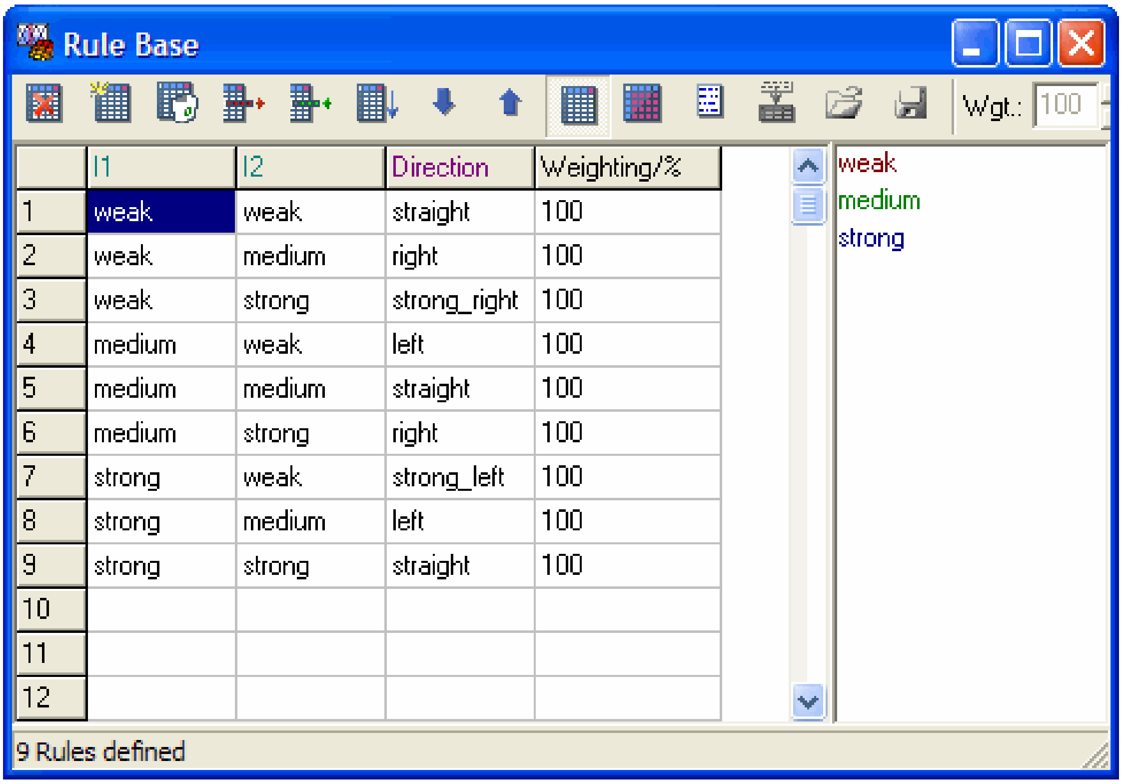
Solution: The linguistic variables can be specified as follows:







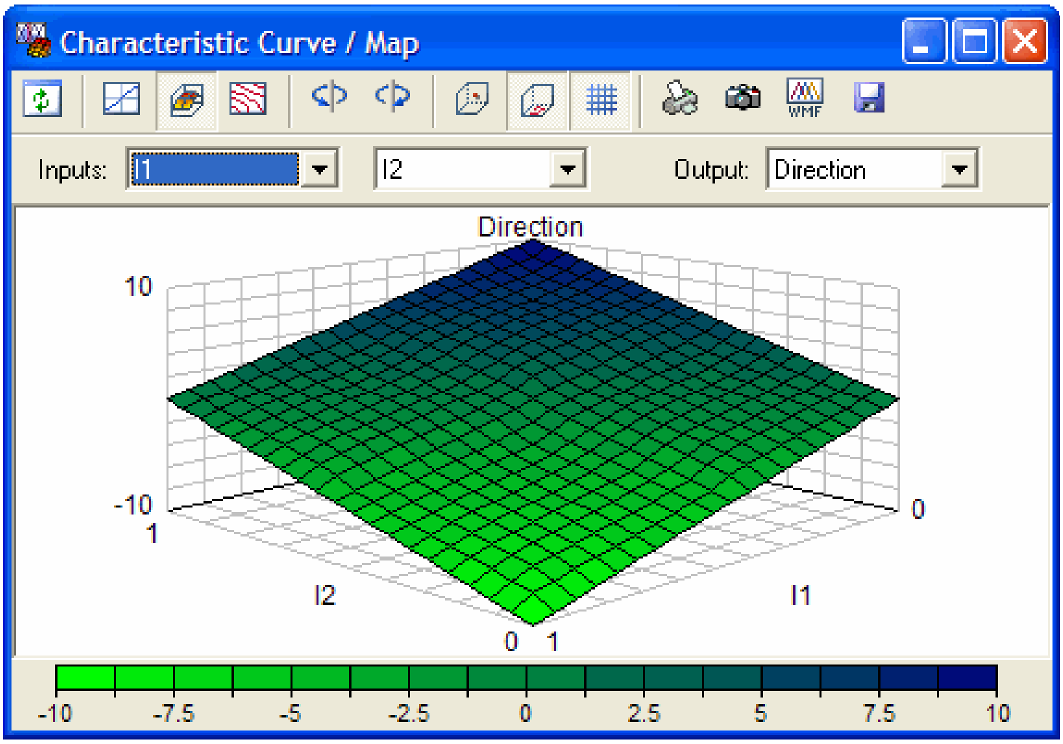
An appropriate rule base is shown by the screenshot below:



	I1	I2	Direction	Weighting/%
1	weak	weak	straight	100
2	weak	medium	right	100
3	weak	strong	strong_right	100
4	medium	weak	left	100
5	medium	medium	straight	100
6	medium	strong	right	100
7	strong	weak	strong_left	100
8	strong	medium	left	100
9	strong	strong	straight	100
10				
11				
12				

To obtain a smooth characteristic map and to use the complete output range the approximated center of gravity defuzzification (center of gravity for singletons) is used. The resulting controller has the following characteristic map:

 © Ingenieurbüro Dr. Kahlert 1991-2006



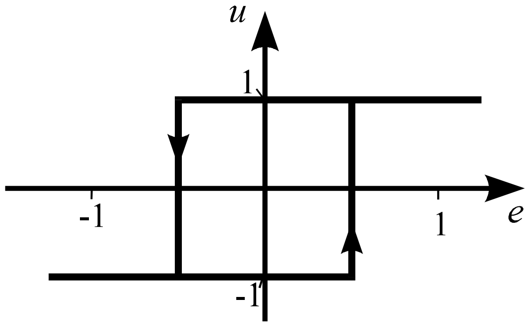
Related files:

AUTOPARK.FUZ

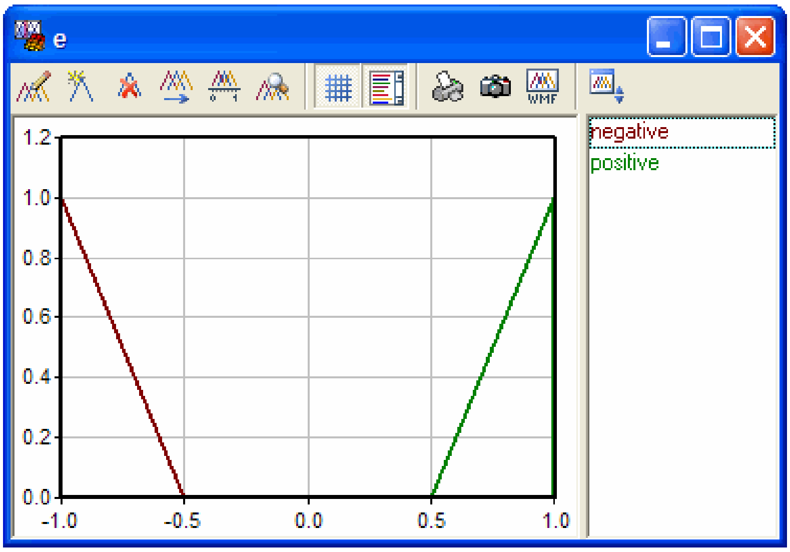
Category V: Fuzzy control

Problem V.1: Fuzzy controller with hysteresis [2]

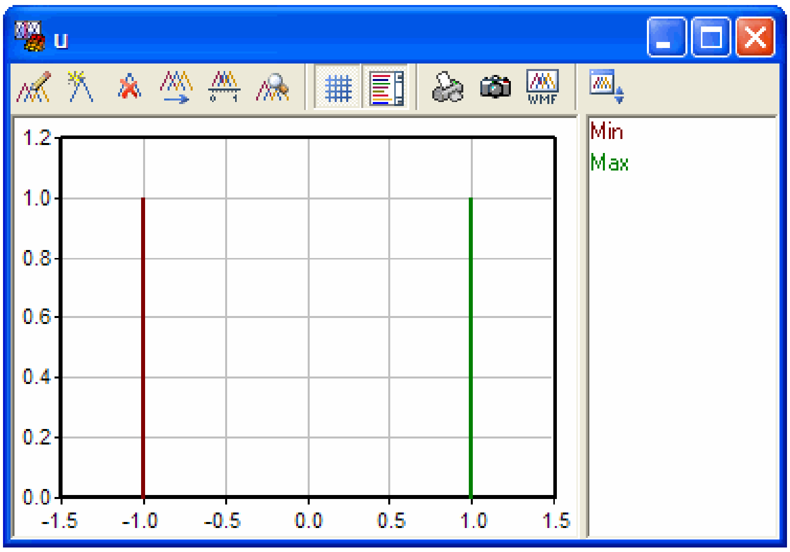
Problem specification: Design a fuzzy controller with input $e(t)$ and output $u(t)$ which has the following characteristic curve with hysteresis:



Solution: To generate the hysteresis no rule must be active for the input range $-0.5 \leq e \leq 0.5$. The fuzzy controller has to keep its last output value in this case. This can be realized e. g. by defining only two input fuzzy sets with a gap in the range mentioned above. The form of the fuzzy sets makes no difference:



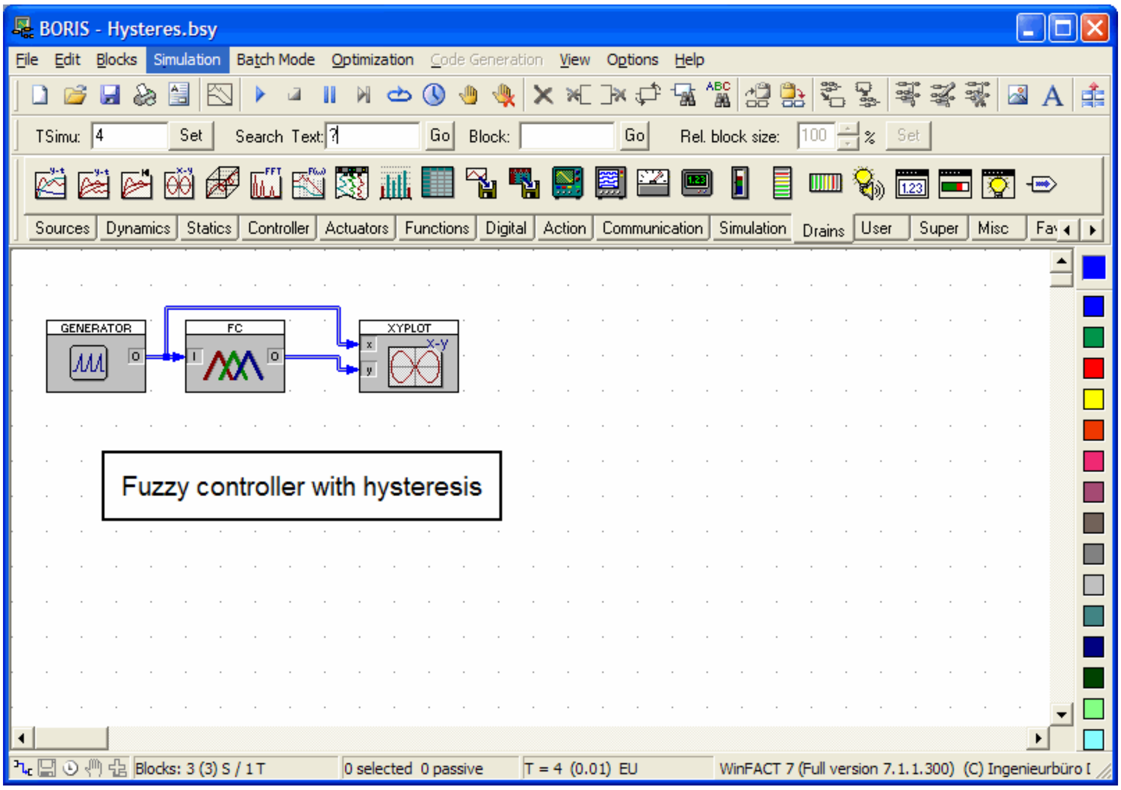
For the output we can select singletons at -1 resp. +1:



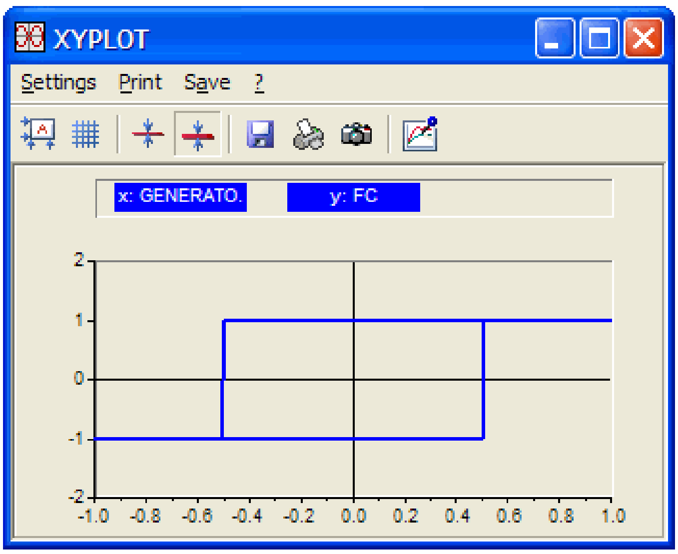
 © Ingenieurbüro Dr. Kahlert 1991-2006

Because always a maximum of one rule is active inference mechanism and defuzzification can be chosen as you like.

For testing our controller we can e. g. use BORIS. The input of the controller is connected to a generator that creates a linear from -1 to 1 increasing and afterwards to -1 decreasing signal. In- and output of the controller are sent to a trajectory plot:



We get the following characteristic curve:



Related files: HYSTERES.FUZ
HYSTERES.BSY

Problem V.2: Comparison: Conventional and Fuzzy P-controller [2]

Problem specification: Given a plant with the transfer function

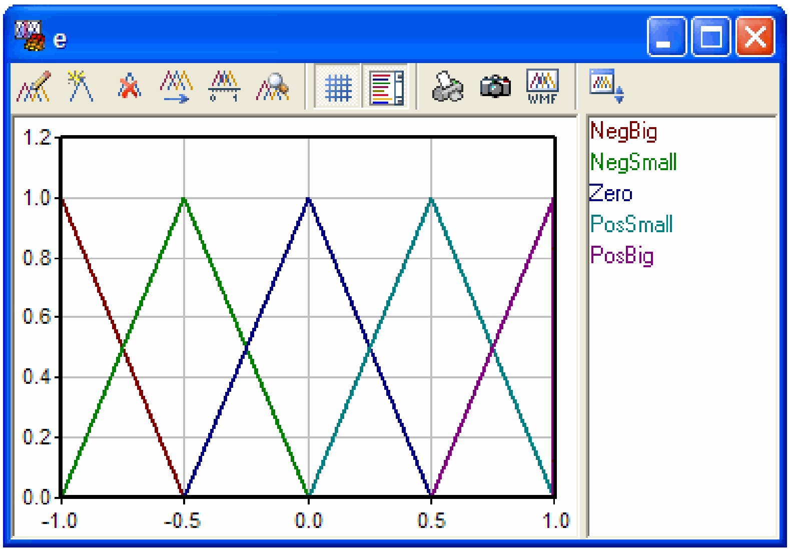
$$G(s) = \frac{1}{(1+s)(1+2s)}.$$

Design a fuzzy P-controller for this plant with five fuzzy sets for input (error) and output (manipulated variable) in standard form and a range of [-1, 1]. Simulate the step response of the resulting closed-loop system up to a simulation time of 10 for the following cases:

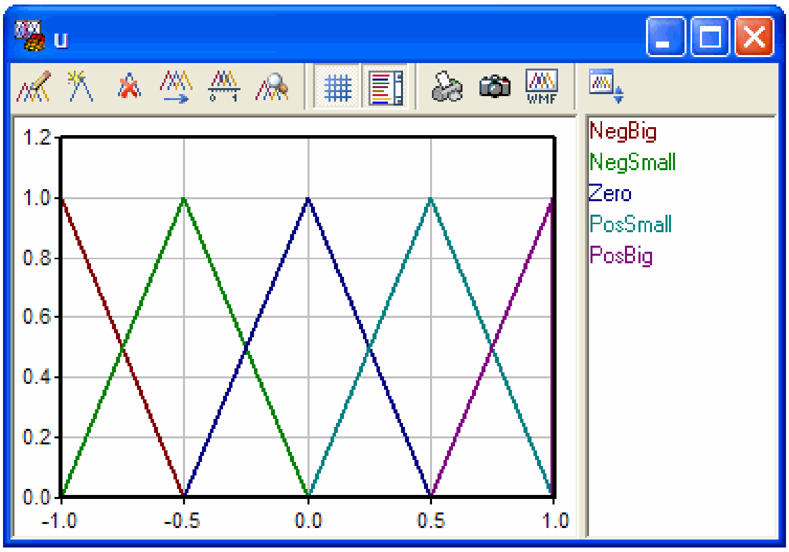
- Max-Min-inference and modified center of gravity method
- Max-Prod-inference and modified center of gravity method
- Max-Min-inference and maximum height method

Compare the results with those determined with a conventional P-controller with gain 1.

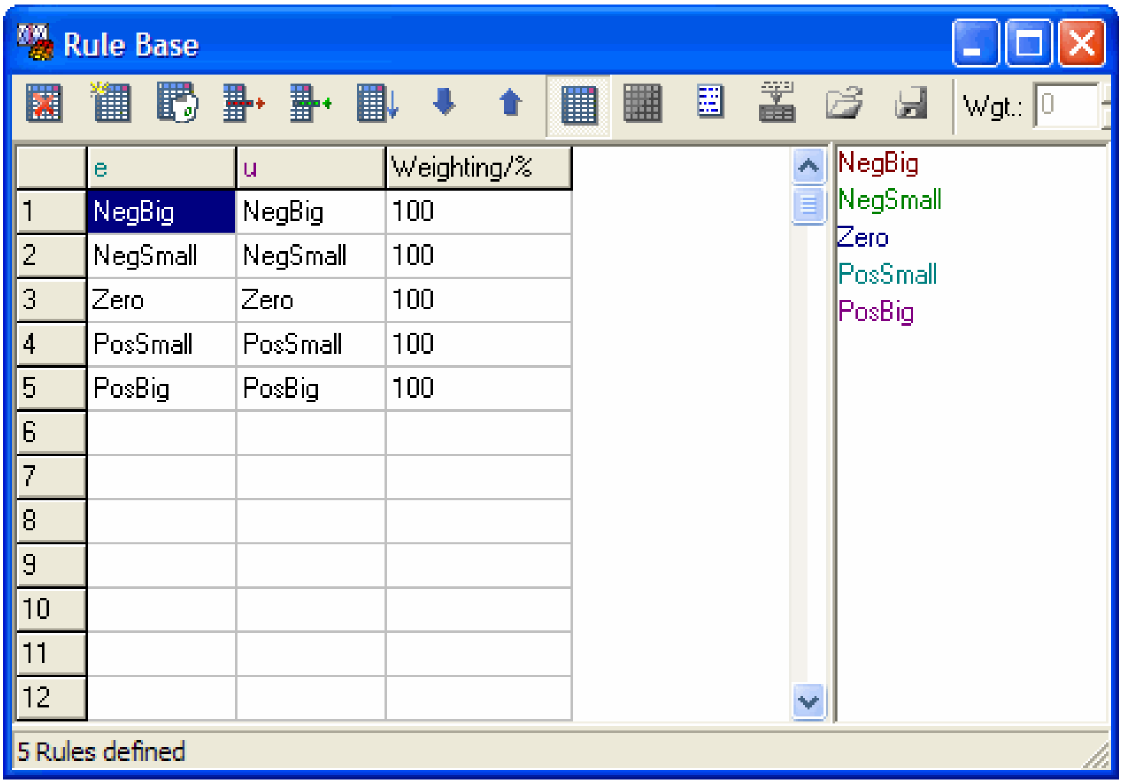
Solution: The fuzzy P-controller can be designed with FLOP. The fuzzy sets are to be defined as follows:



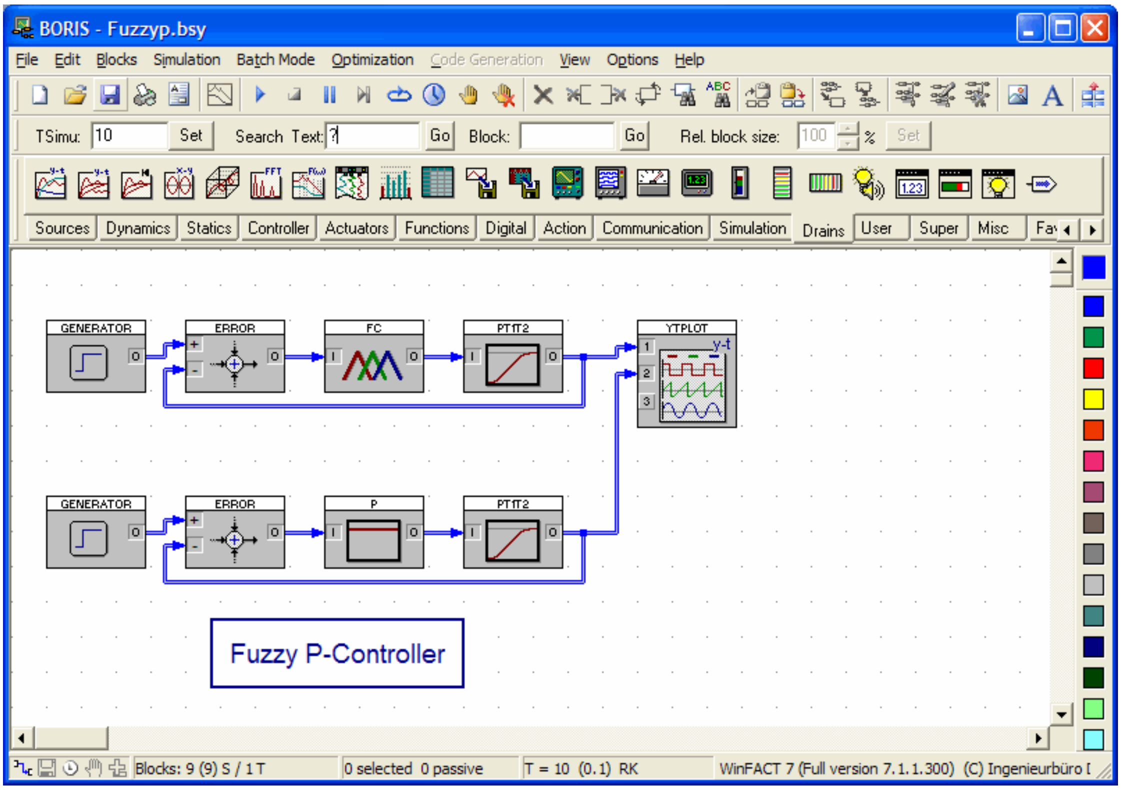
 © Ingenieurbüro Dr. Kahlert 1991-2006



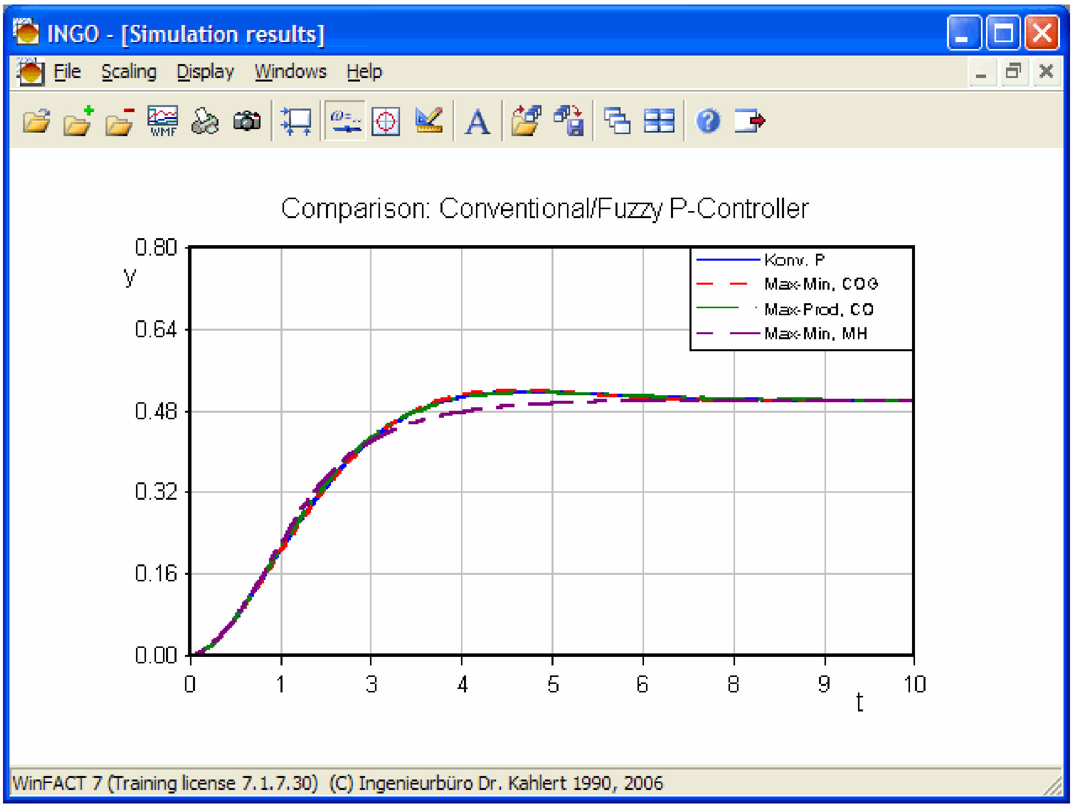
The rule base has the following structure:



The simulation is executed with BORIS. Both closed-loop systems (with Fuzzy resp. conventional P-controller) can be simulated simultaneously:



The results are saved and compared with INGO. We get the following step responses:



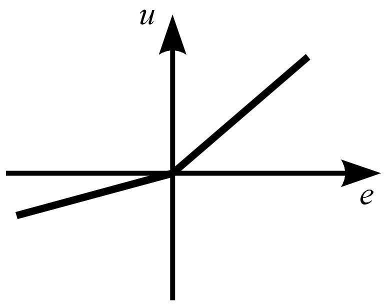
 © Ingenieurbüro Dr. Kahlert 1991-2006

We can clearly recognize that the fuzzy controller with center of gravity defuzzification and max-min- resp. max-prod inference leads nearly exactly to the same step response of the closed-loop system as the conventional P-controller. Only if the maximum height method is used for defuzzification the fuzzy controller leads to a worse dynamic behaviour because in this case the controller has a step-sized characteristic curve (multi-relay characteristic).

Related files: FUZZYP.FUZ
FUZZYP.BSY

Problem V.3: Split range fuzzy controller

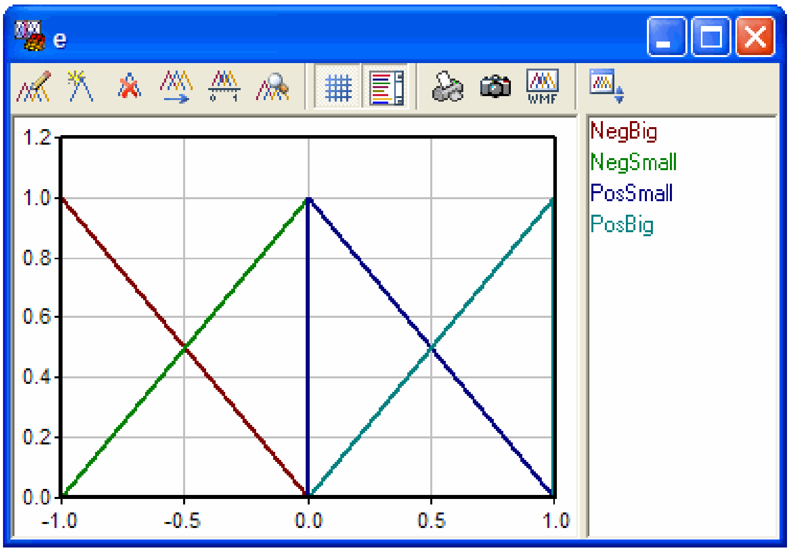
Problem specification: Design a fuzzy controller with input *e* and output *u* which has different gains for positive and negative input values, i. e. which has a characteristic curve of the following form:



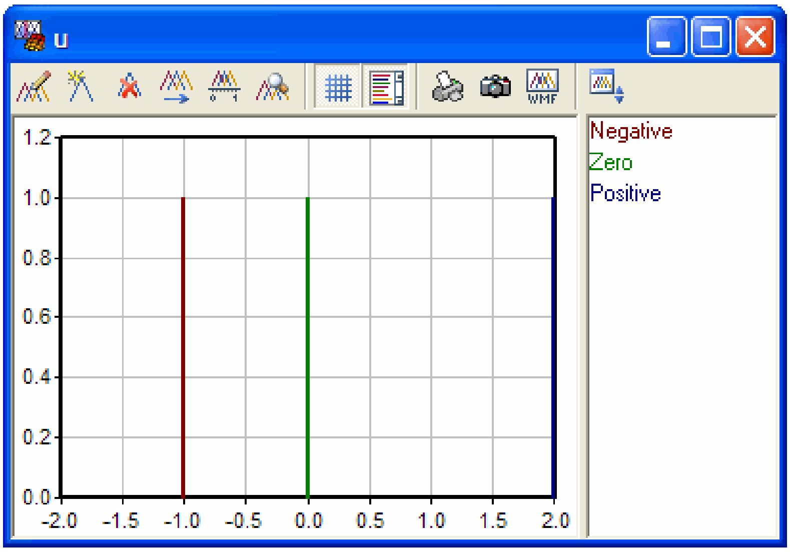
The gain for positive input values is to be 1, the gain for negative values 2.

Solution: One possible solution is as follows:

The fuzzy sets for *e* are chosen as follows:

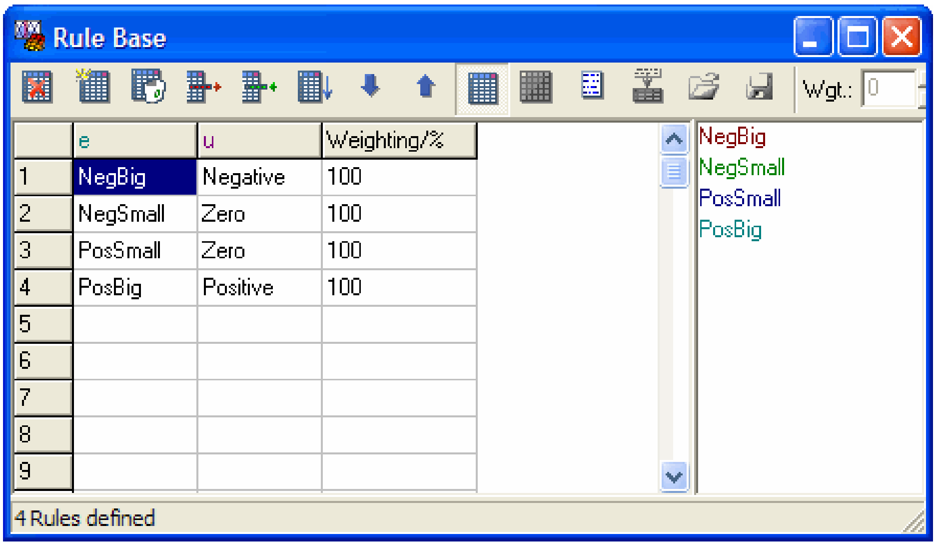


The fuzzy sets for *u* are selected as singletons at -1, 0 and 2:



Finally the rule base has the following structure:

 © Ingenieurbüro Dr. Kahlert 1991-2006



The inference mechanism is max-min, the defuzzification is executed by the center of gravity method for singletons.

Related files:

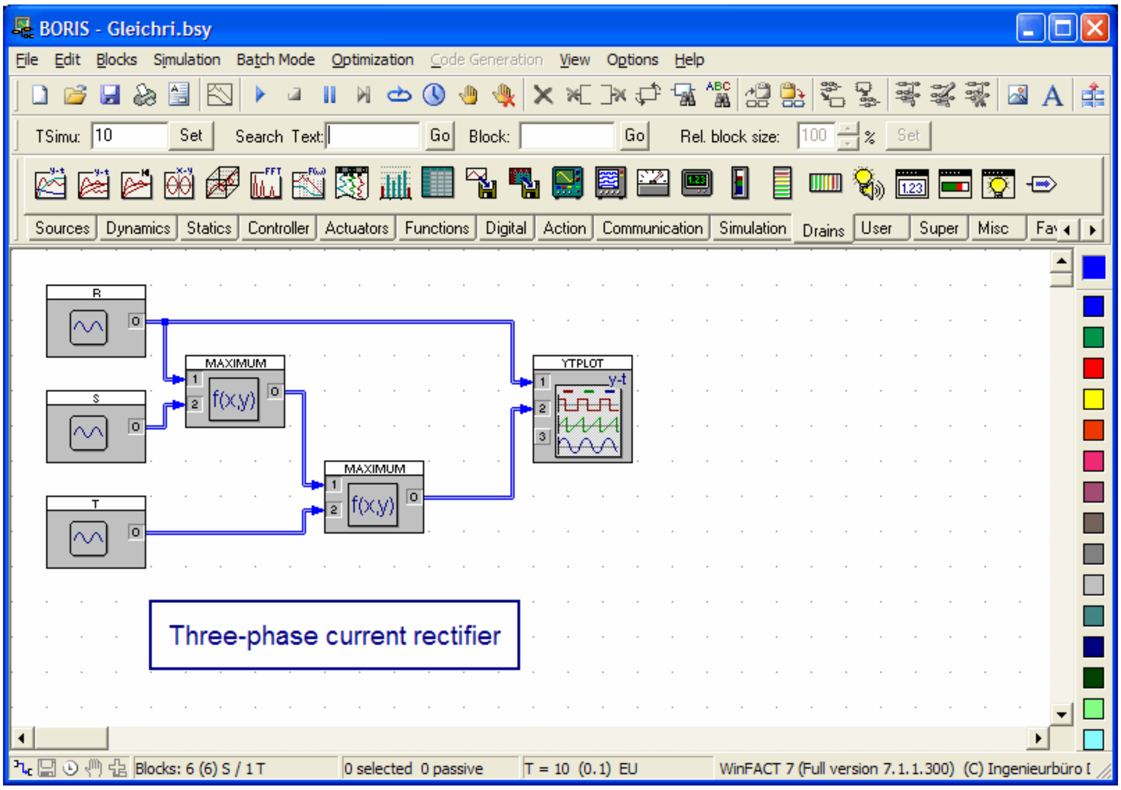
SPLITRAN.FUZ

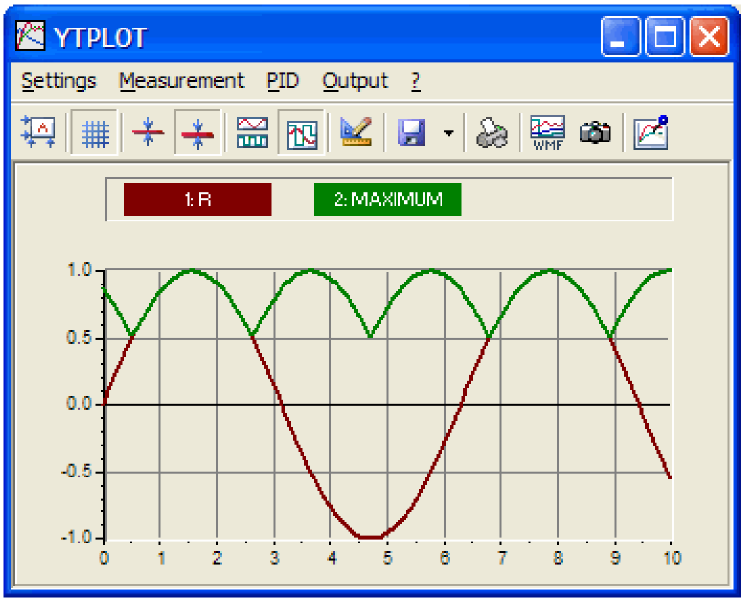
Category VI: Measurement technology

Problem VI.1: Three-phase current rectifier

Problem specification: Design a rectifier for a three-phase current.

Solution: The commutation of the three phases is realized with two function blocks which determine the maximum of two input signals each. The screenshots below illustrate the simulation structure and the resulting signal.





Related files:

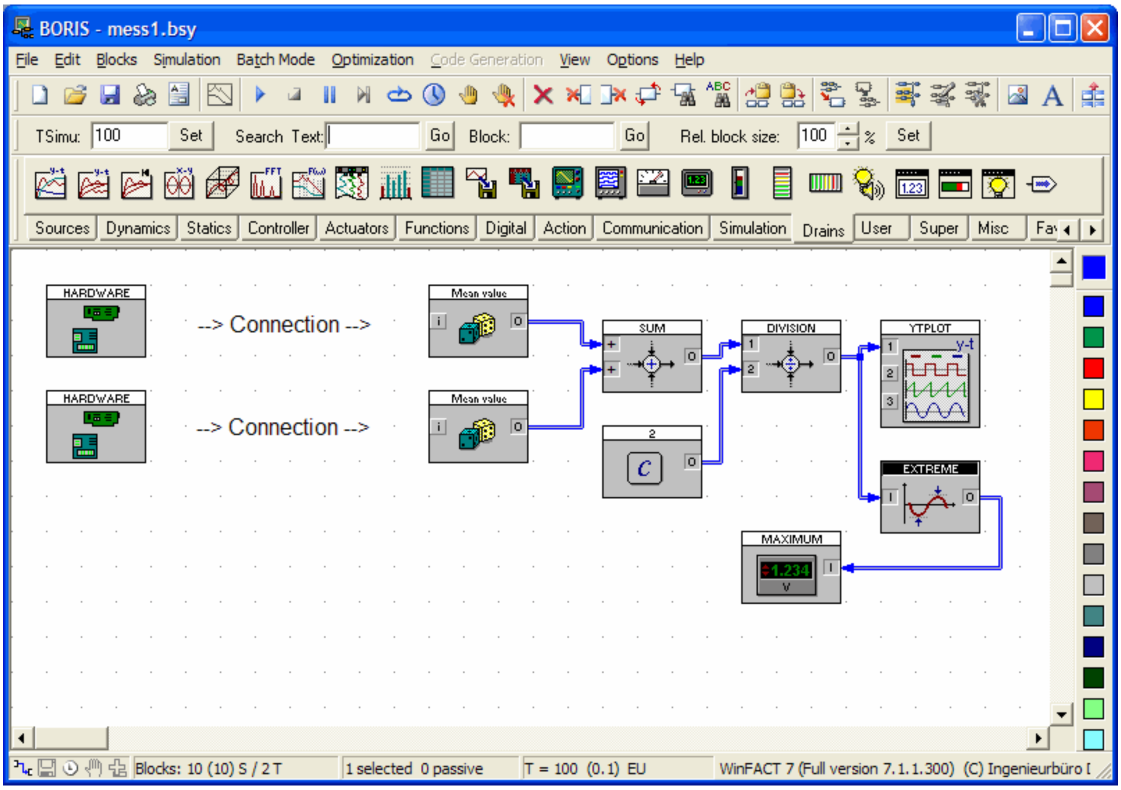
GLEICHRI.BSY

© Ingenieurbüro Dr. Kahlert 1991-2006

Problem VI.2: Signal data acquisition

Problem specification: Two analog signals are to be read via an A/D converter card; the mean value of both signals is to be displayed in form of a time response. Furthermore the maximum is to be displayed on a digital meter.

Solution: The problem can be solved by using the following simulation structure:



Related files:

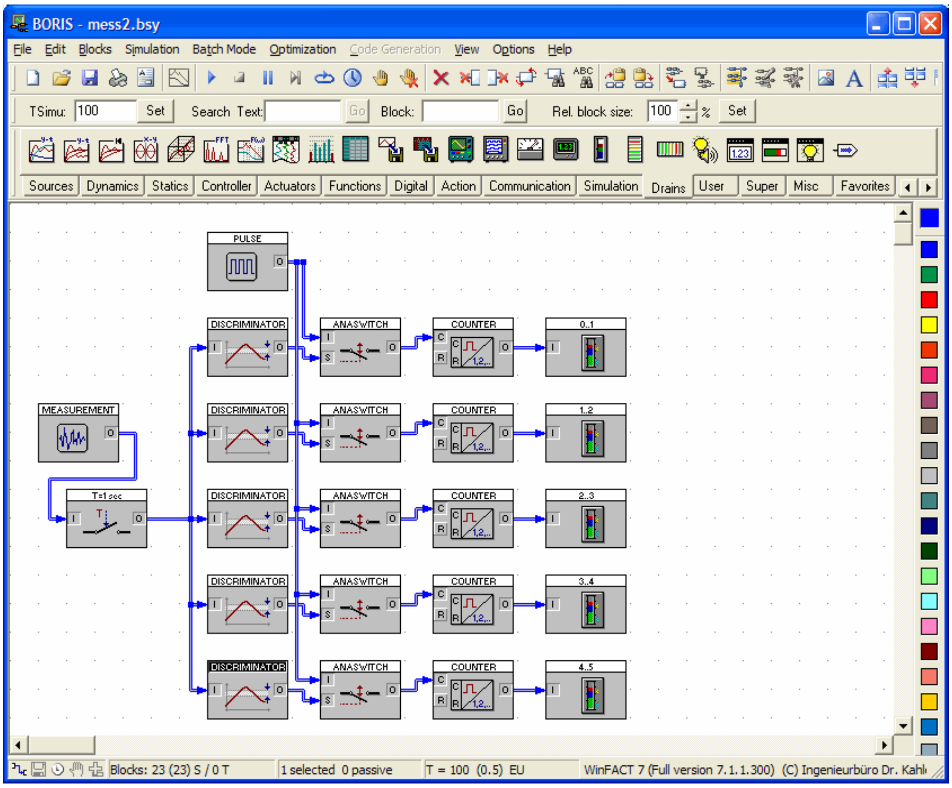
MESS1.BSY

Problem VI.3: Statistical evaluation with bar graphs

Problem specification: A stochastical analog signal is to be sampled with a sample time $\Delta T = 1$. It is known that this signal on an average takes all values between 0 and 5 with different but known frequency. To get an overview whether the signal is sampled correctly the signal range is divided into five classes and the number of samples within each class shall be displayed by bar graphs. Design a simulation structure for this purpose which works without integrator.

© Ingenieurbüro Dr. Kahlert 1991-2006

Solution: The classification is realized by discriminators connected to forward counters. Because the counters are triggered by edges the static discriminator levels are changed to pulse form with the help of a relay and a pulse generator. To let the circuit work correctly the simulation step size must be less than or equal to the half of the sample time, i. e. maximum 0.5 here.



The structure above is only one possible solution. Another solution working with less blocks eliminates the relays and multiplies the sampled signal behind the S/H-element with the signal of the pulse generator. In this case the amplitude of the pulse generator must be 1.

Related files:

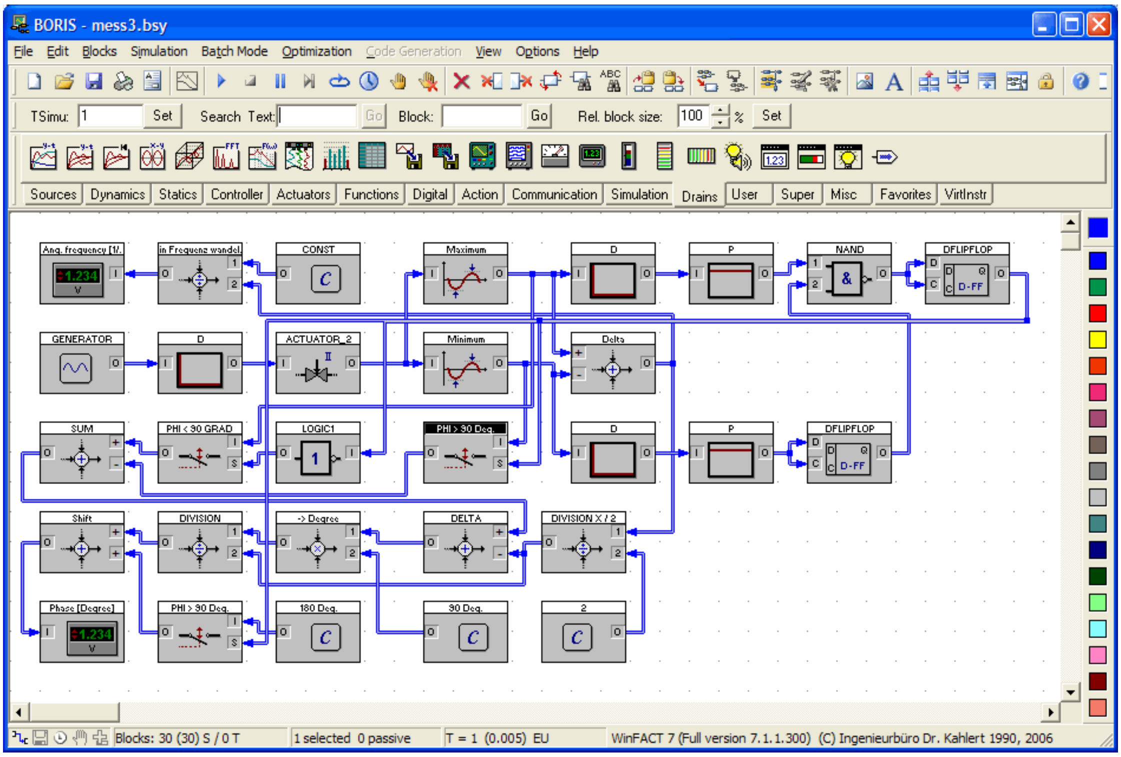
MESS2.BSY

 © Ingenieurbüro Dr. Kahlert 1991-2006

Problem VI.4: Measuring phase and frequency

Problem specification: It is very simple to display frequency and phase shift of a sinusoid signal by using a scope. The disadvantage in doing so is that the time base of the scope has to be adjusted any time the frequency of the input signal changes. In this Problem a solution is searched for which displays frequency and phase on a digital meter. Design a simulation structure which fulfills this requirement.

Solution: By incrementing resp. decrementing during the first half wave (ACTUATOR_2) we receive a maximum and minimum value containing the complete information we need. The offset of this triangular signal can be used to determine the phase, the peak-to-peak value for frequency recalculation. The screenshot below shows the simulation structure.



Related files:

MESS3.BSY

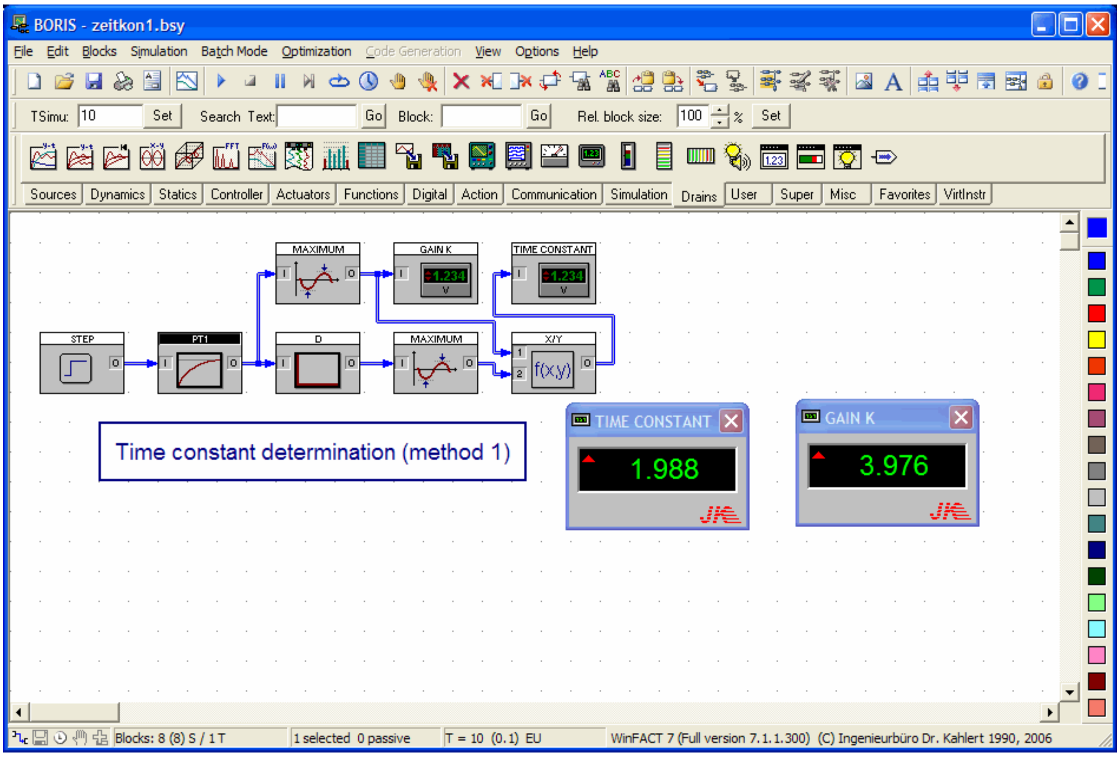
Problem VI.5: Determination of a PT_1-time constant

Problem specification: The time constant of a PT1-element with the transfer function

$$G(s) = \frac{K}{1+Ts}$$

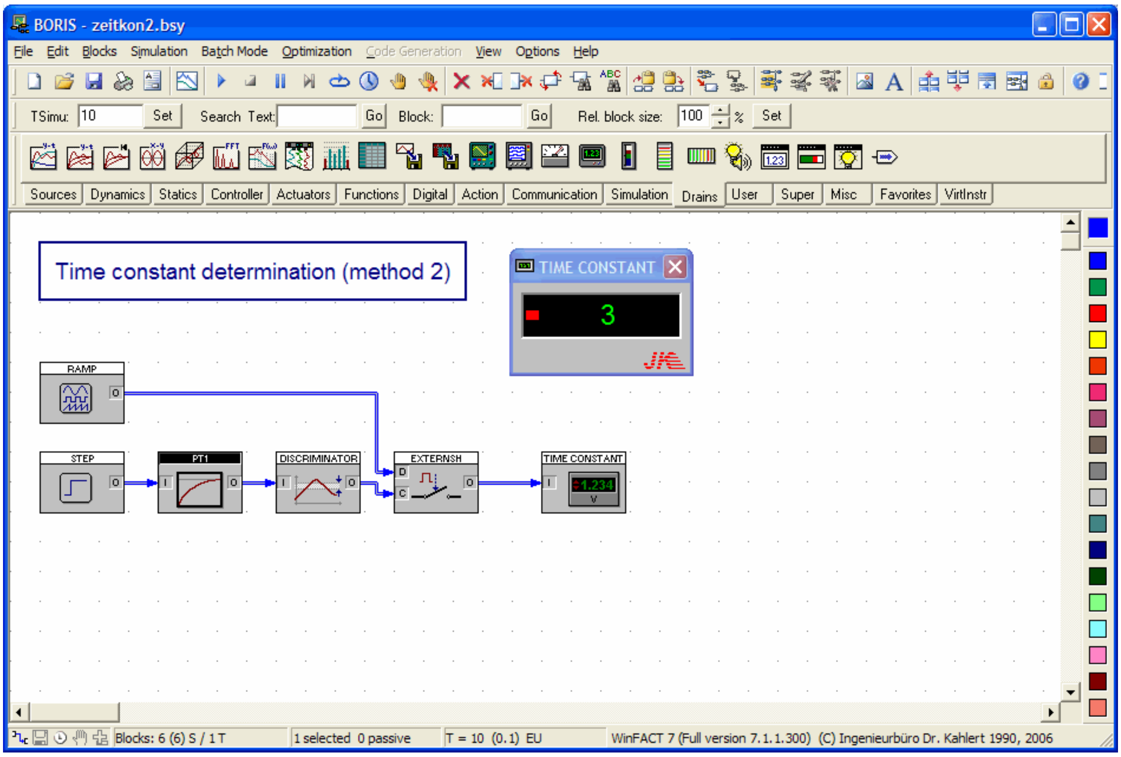
is to be determined based on the step response of the element.

Solution: Different solutions do exist. One variant is to differentiate the step response and determine the maximum of this function, i. e. the initial gradient of the step response. This value is K/T. Thus the gain K also has to be determined to let this method work correctly. This can be done by determining the maximum of the step response as shown below.



The second method is to determine the time when the step response reaches the value $K(1-1/e)$:

© Ingenieurbüro Dr. Kahlert 1991-2006



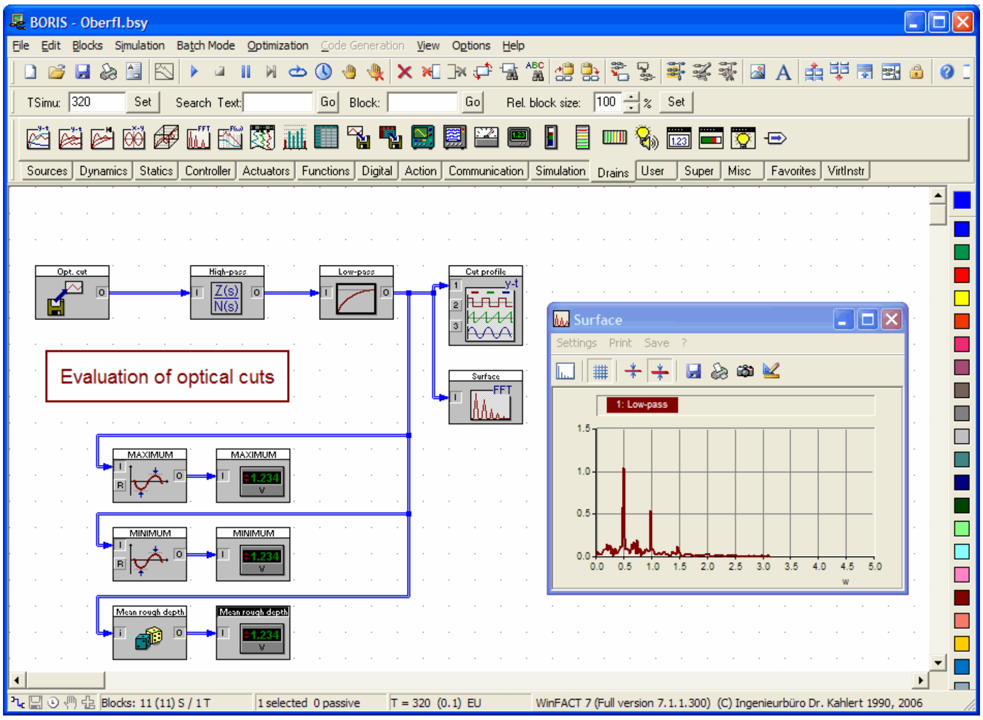
This method only works if the gain *K* of the PT1-element is already known beacuse it has to be considered in the discriminator.

Related files: ZEITKON1.BSY
ZEITKON2.BSY

Problem VI.6: Evaluation of optical cuts [11]

Problem specification: A simulation circuit is to be found which analyzes optical cuts saved in SIM-files. First the cut profile has to be sent through a band-pass filter; afterwards the amplitude spectrum of the cut profile as well as minimum, maximum and mean rough depth have to be determined.

Solution: The band-pass filter is realized by a TRANSFCT-block combined with a PT1-element. The structure is as follows:

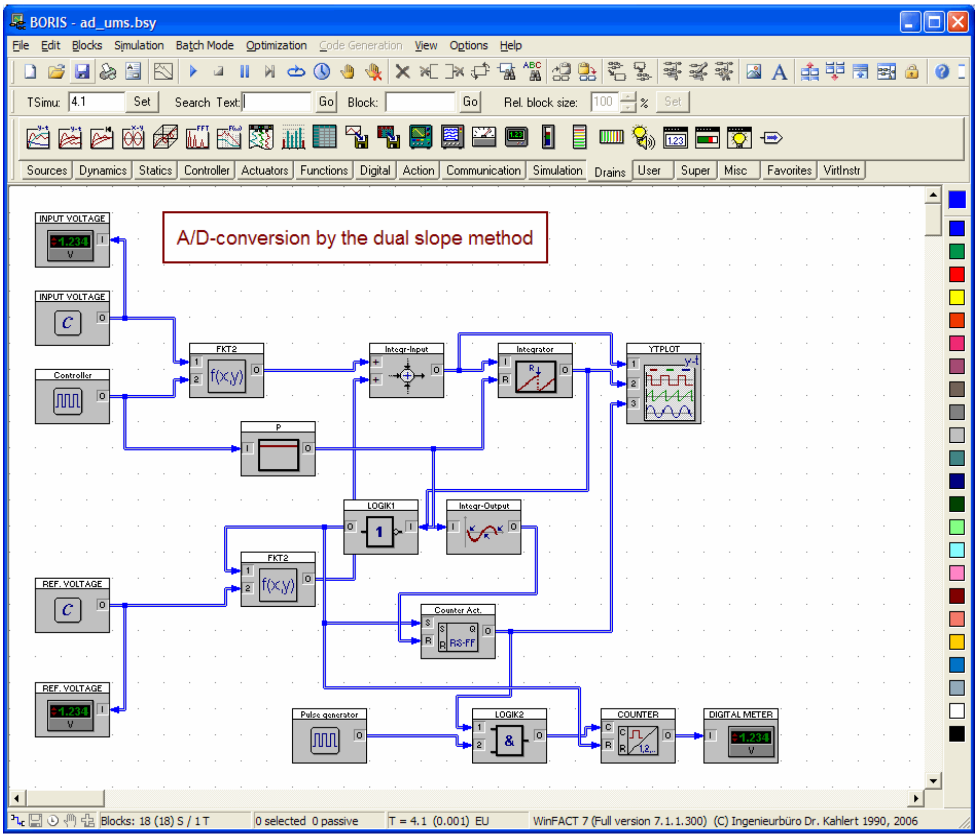


Related files: OBERFL.BSY
OBERFL.SIM

Problem VI.7: A/D-conversion by the dual slope method [11]

Problem specification: A BORIS simulation structure is to be found which demonstrates an A/D-conversion based on the dual slope principle.

Solution: The screenshot below shows an appropriate simulation structure.



Related files:

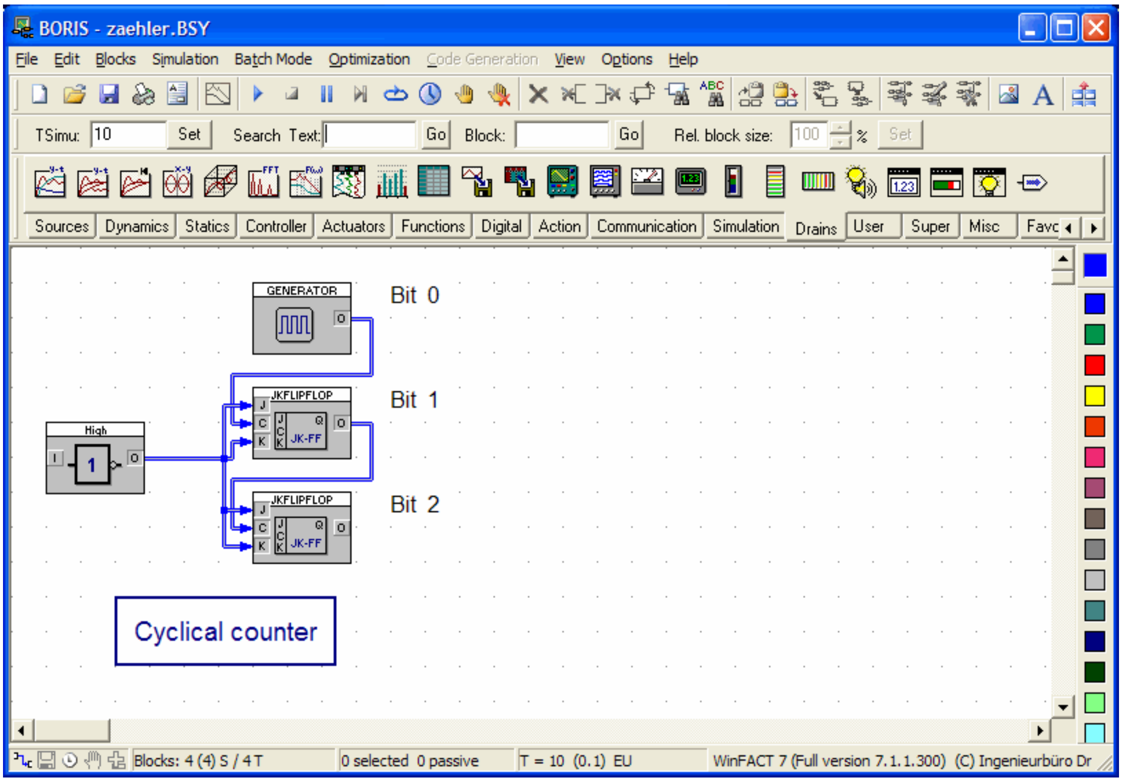
AD_UMS.BSY

Category VII: Digital technology

Problem VII.1: Cyclic dual counter

Problem specification: For test purposes a digital net with three inputs shall take all possible states. Design a digital circuit that fulfills this requirement using only one generator.

Solution: The generator is used as a clock generator und thus must be set to the pulse mode. Its output signal is connected to the clock input of a JK-flip flop working as a frequency divider (J- and K-input set to HIGH). The flip flop output is connected to a second JK-flip flop which also works as a frequency divider.



Related files:

ZAEHLER.BSY

Problem VII.2: 2 bit adder

Problem specification: An adder for 2 positive 2-bit-binary numbers has to be designed. Consider all possible transfers. The result has to be a 3-bit-binary number. Afterwards change the circuit in such a way that the result is representd as a 2-bit-binary number with overflow flag.

Solution: The first 2-bit-number maybe $A_2 A_1$, the second $B_2 B_1$. Then the sum $C_2 C_1 = A_2 A_1 + B_2 B_1$ is calculated as follows:

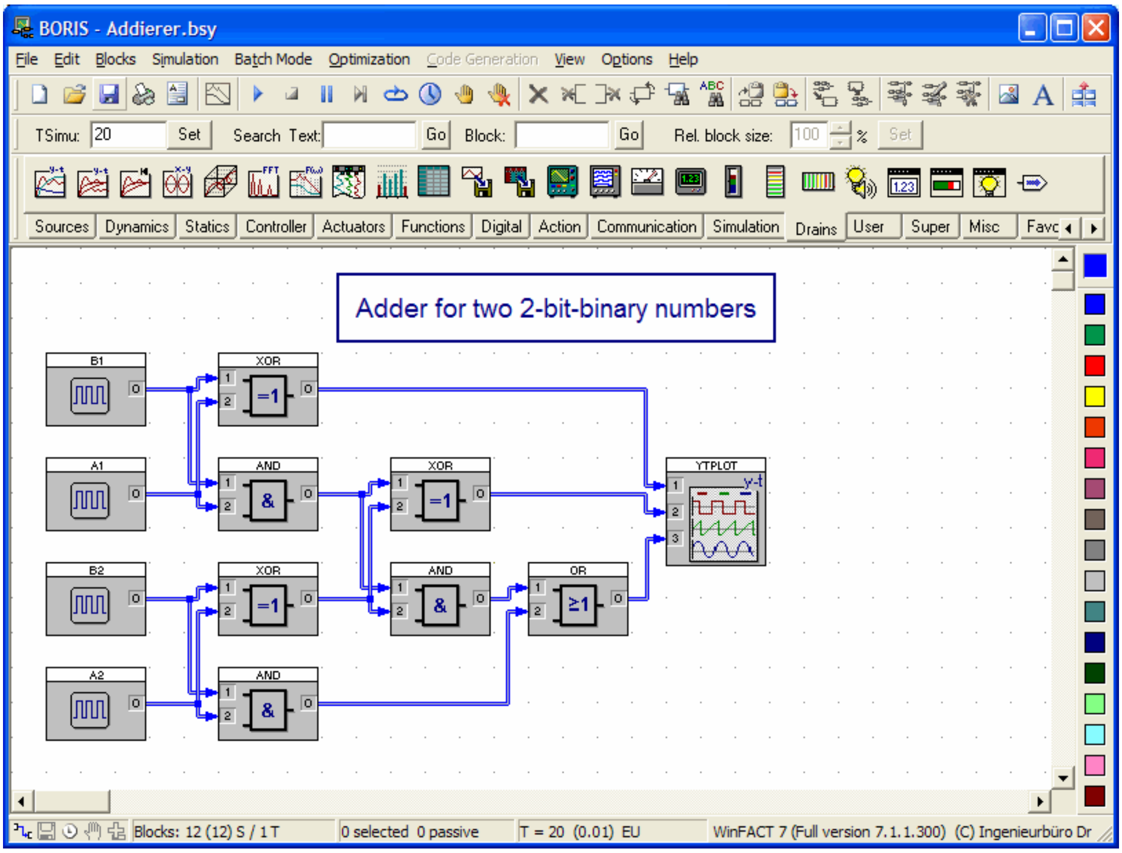
© Ingenieurbüro Dr. Kahlert 1991-2006

$$C_1 = A_1 \neq B_1$$
$$C_2 = (A_1 \wedge B_1) \neq (A_2 \neq B_2)$$

The overflow flag is given by:

$$C_3 = (A_2 \wedge B_2) \vee \left((A_2 \neq B_2) \wedge (A_1 \wedge B_1)\right)$$

Thus we get the following simulation structure:



Related files:

ADDIERER.BSY

Problem VII.3: Code conversion Gray code -> Dual code

Problem specification: A control unit that gets information over a sensor with a 3-bit gray code is to be connected to the PC. The PC has to visualize the current state of the sampling process in dual code. Design a corresponding conversion circuit.

Solution: It is $g_2 g_1 g_0$ the value in gray code and $d_2 d_1 d_0$ the corresponding dual value. The table below shows the conversion rules:

Gray			Dual		
g_2	g_1	g_0	d_2	d_1	d_0
0	0	0	0	0	0
0	0	1	0	0	1
0	1	1	0	1	0
0	1	0	0	1	1
1	1	0	1	0	0
1	1	1	1	0	1
1	0	1	1	1	0
1	0	0	1	1	1

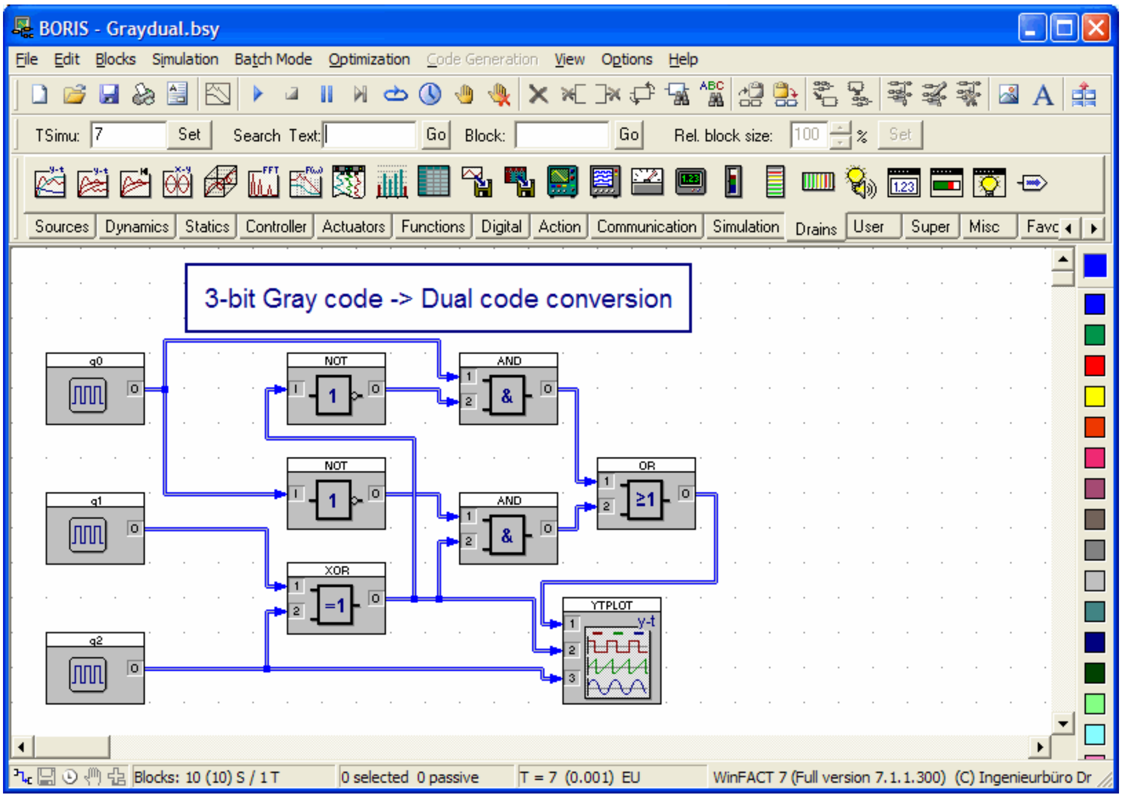
From this table the following logic can be concluded:

$$d_2 = g_2$$

$$d_1 = g_1 \neq g_2$$

$$d_0 = \left(g_0 \wedge (g_1 = g_2)\right) \vee \left(\overline{g_0} \wedge (g_1 \neq g_2)\right)$$

This corresponds to the following system structure:



Related files:

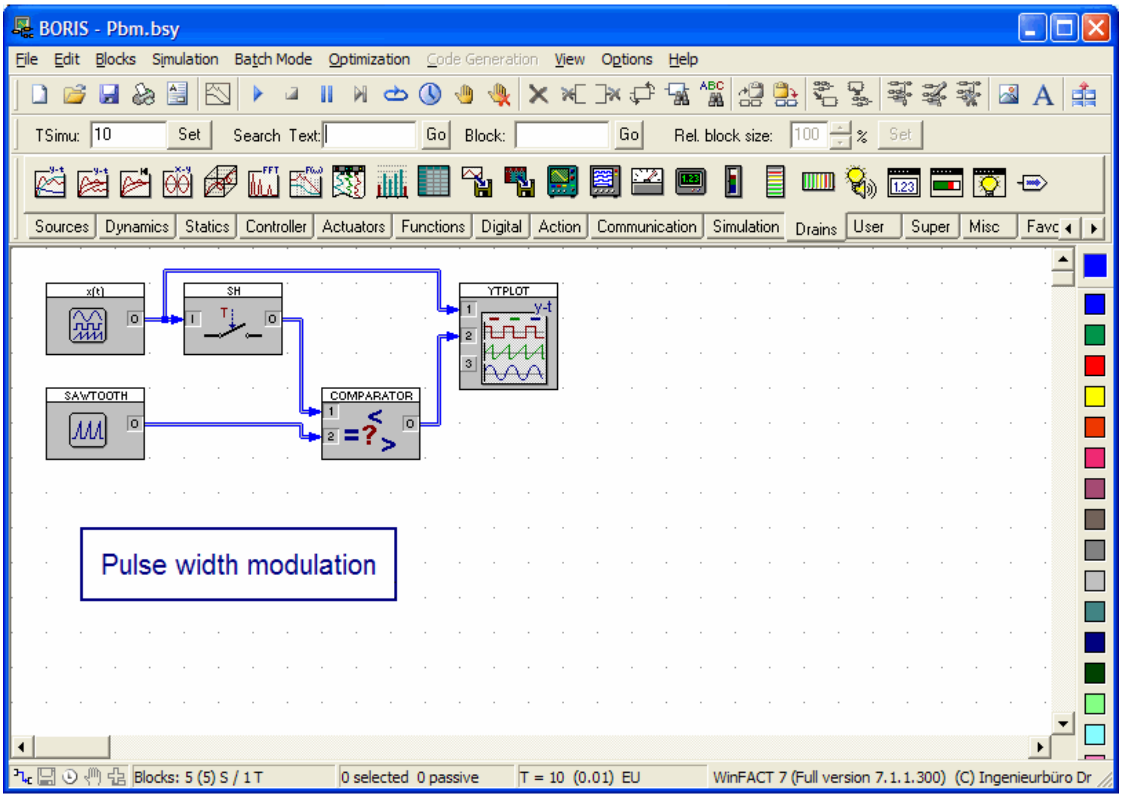
GRAYDUAL.BSY

© Ingenieurbüro Dr. Kahlert 1991-2006

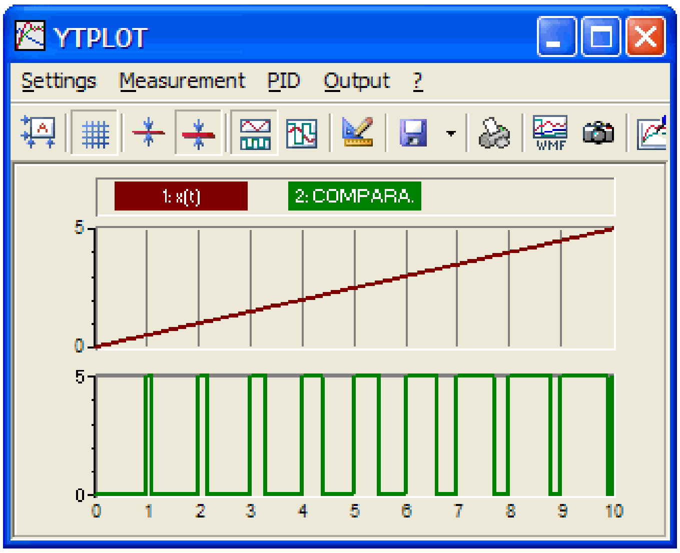
Problem VII.4: Pulse width modulation

Problem specification: A structure is to be designed which converts an analog input signal $x(t)$ with an amplitude between 0 and 5 to a pulse width modulated signals with a frequency of 1 Hz.

Solution: The input signal is sampled with 1 Hz and connected to a comparator which compares it with the signal of a saw-tooth generator which also works with 1 Hz. The greater the amplitude of the analog signal, the logner the output of the comparator keeps HIGH-level.



The screenshot below shows the PWM signal for an input signal which increases linear from 0 to 5:



Related files:

PBM.BSY

Problem VII.5: Digital filter of second order [13]

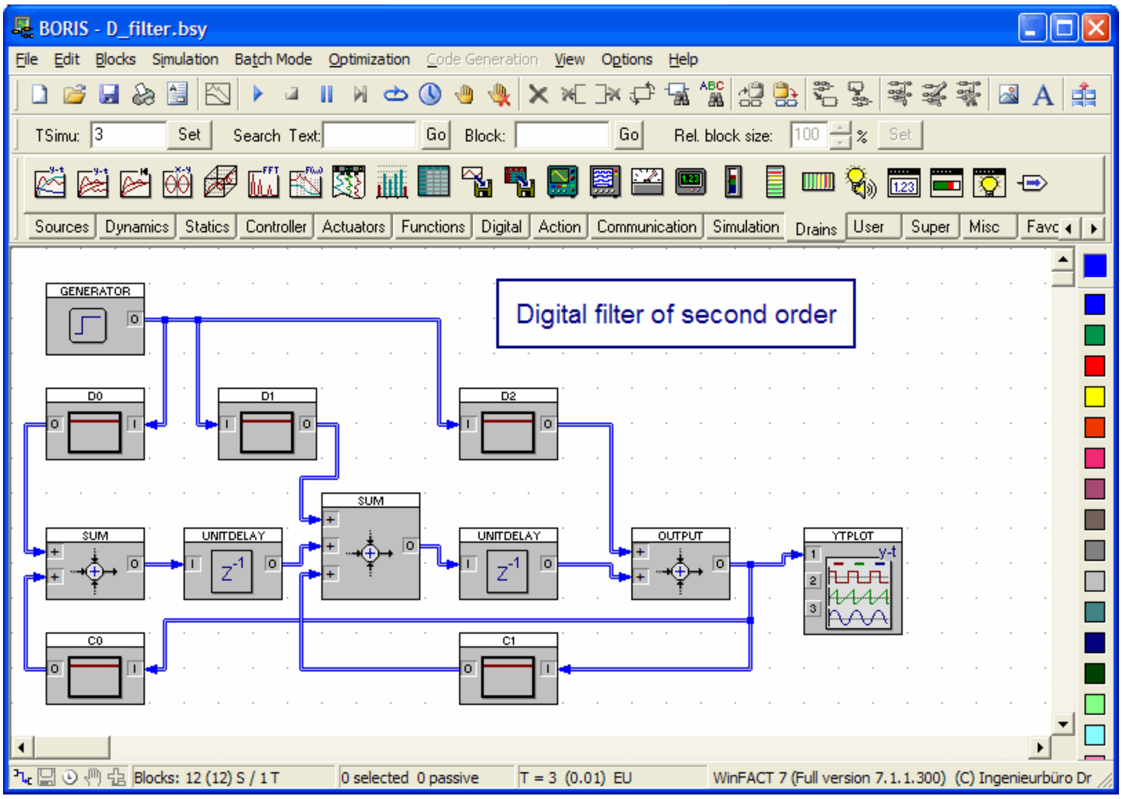
Problem specification: Determine the step response of a digital filter with the *z*-transfer function

$$G(z) = \frac{0.0582 + 0.1164z^{-1} + 0.0582z^{-2}}{1 - 1.4409z^{-1} + 0.6737z^{-2}}$$

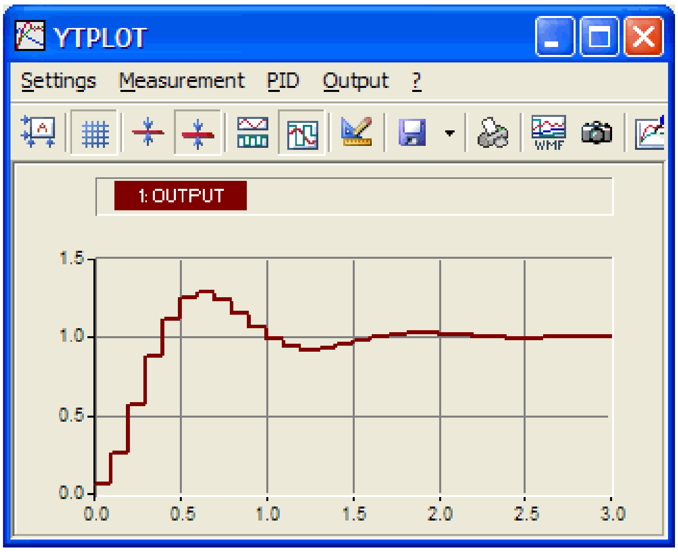
for a sampling time of 0.1 Use a simulation step size of 0.01 and a simulation length of 3.

Solution: The digital filter is realized using unit delays z^{-1} as follows:

© Ingenieurbüro Dr. Kahlert 1991-2006



You get the following step response:



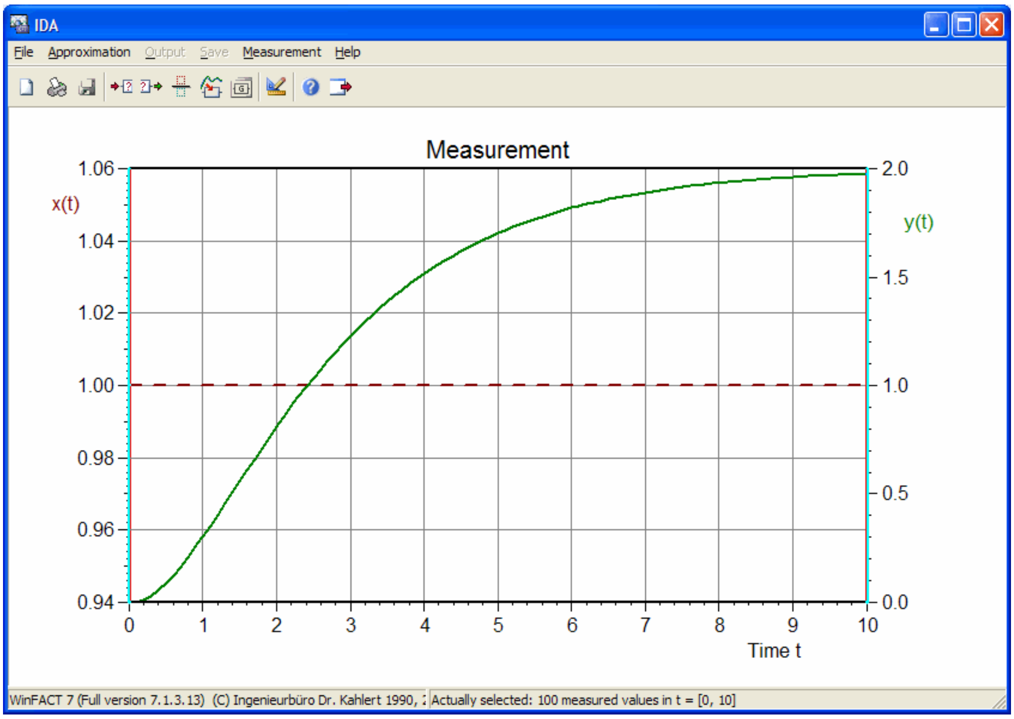
Related files:

D_FILTER.BSY

Category VIII: System identification

Problem VIII.1: Identification based on the step response

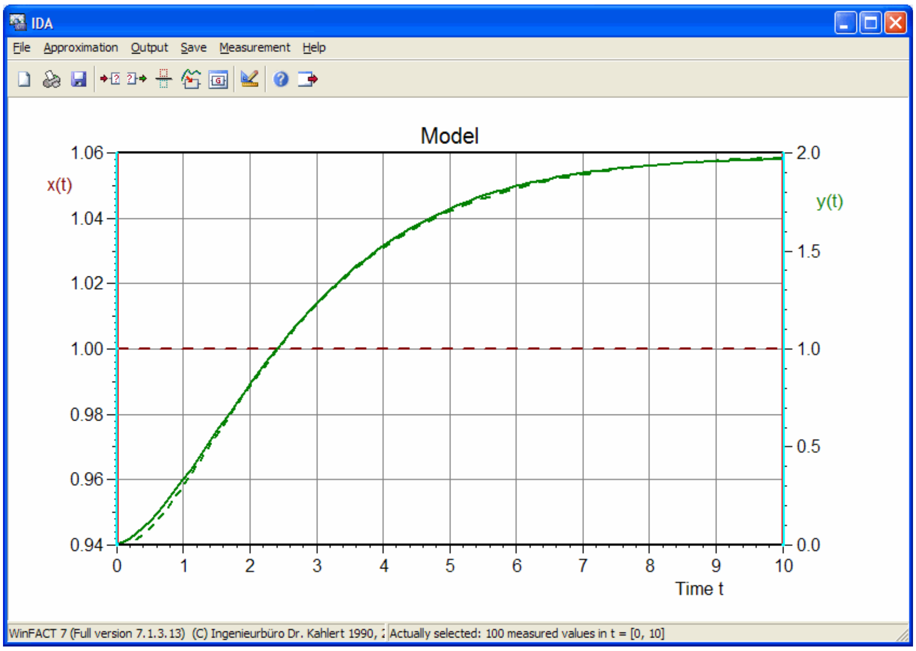
Problem specification: The step response of a plant with input $x(t)$ and output $y(t)$ is measured as follows:



The input signal is represented by a dashed line, the output signal by a solid line. Determine the transfer function of the corresponding model.

Solution: The model is determined by using LISA. The numerator order is chosen as $m = 0$, the denominator order as $n = 2$. We get the following result:

 © Ingenieurbüro Dr. Kahlert 1991-2006



The corresponding transfer function is

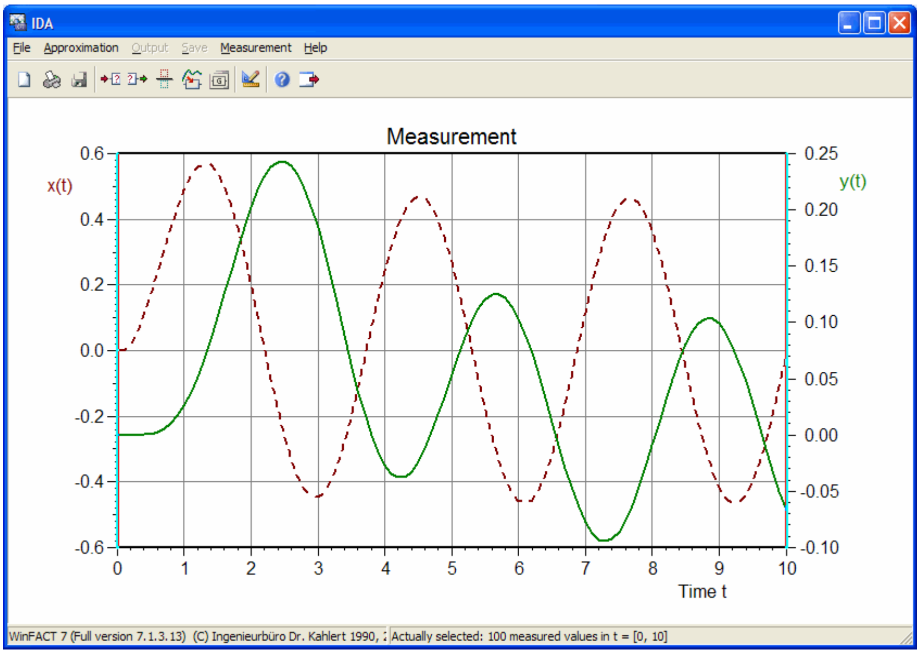
$$G(s) = \frac{0.873}{s^2 + 0.441s + 1.282}.$$

Related files: SPRUNG_X.SIM
SPRUNG_Y.SIM

Problem VIII.2: Identification with harmonic input signals

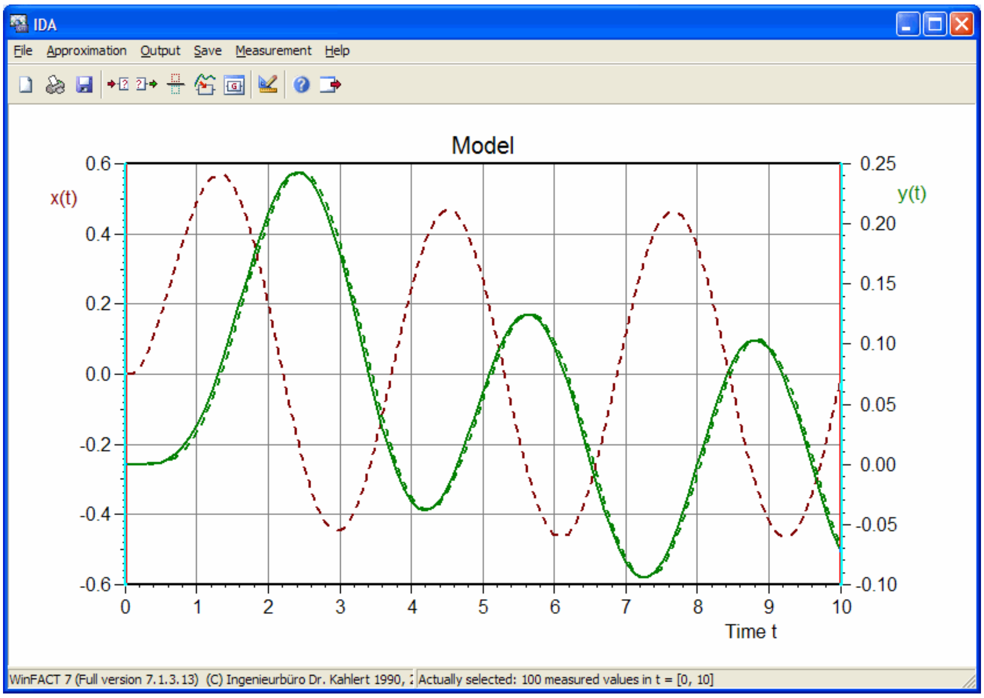
Problem specification: For a plant the following courses of input signal $x(t)$ and output signal $y(t)$ are measured:

© Ingenieurbüro Dr. Kahlert 1991-2006



The input signal is represented by a dashed line, the output signal by a solid line. Determine the transfer function of the corresponding model.

Solution: The model is determined by using LISA. The numerator order is chosen as $m = 0$, the denominator order as $n = 3$. We get the following result:



The corresponding transfer function is

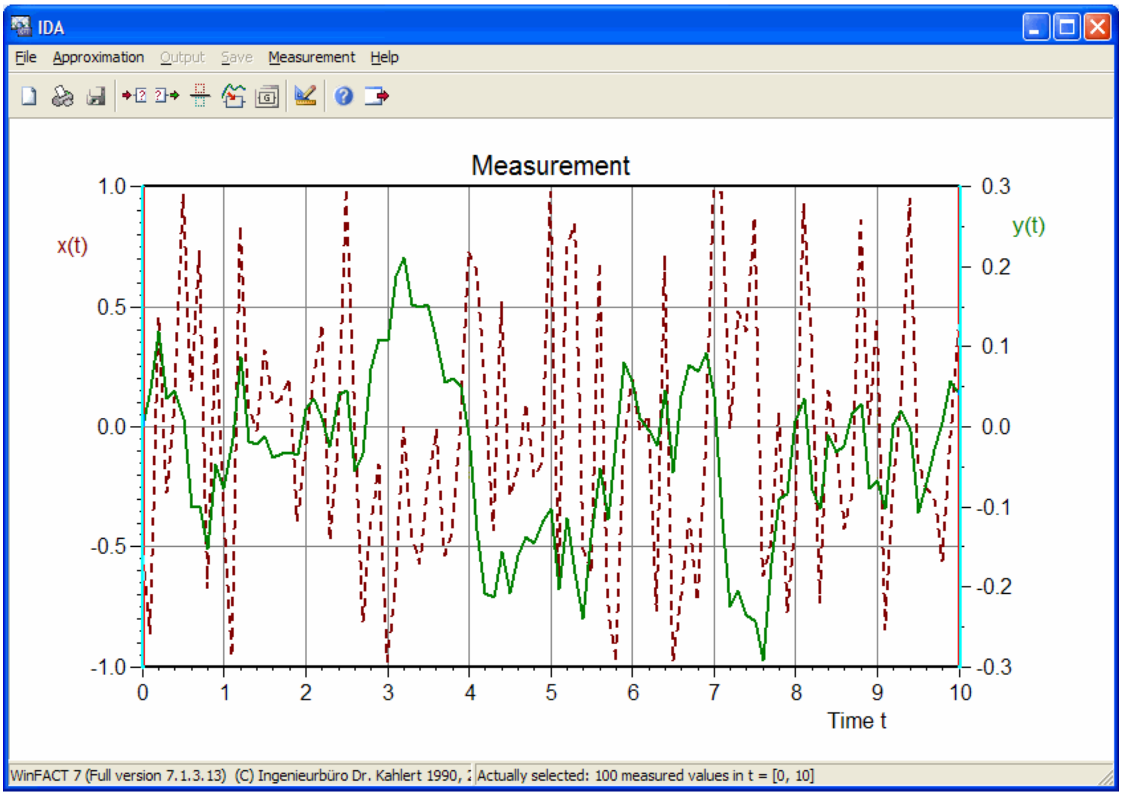
© Ingenieurbüro Dr. Kahlert 1991-2006

$$G(s) = \frac{9.02}{s^3 + 9.83s^2 + 18.08s + 9.01}$$

Related files: SINUS_X.SIM
SINUS_Y.SIM

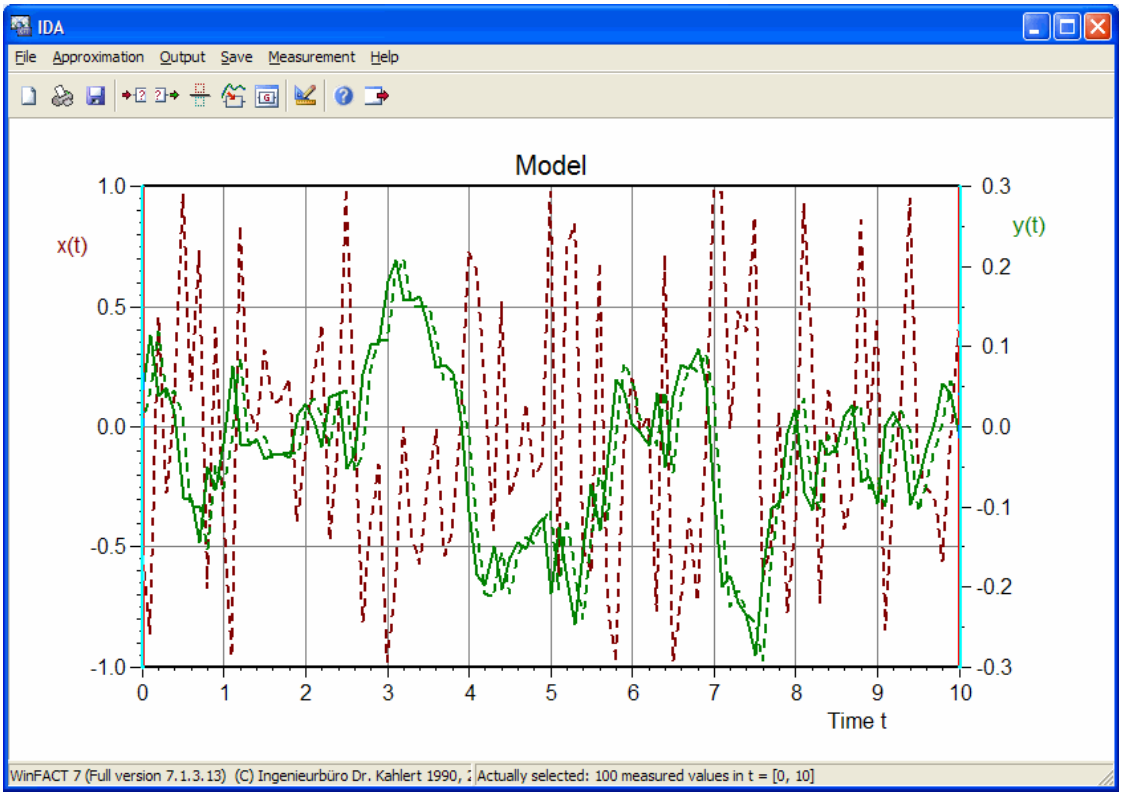
Problem VIII.3: Identification from noisy signals

Problem specification: For a plant the following courses of input signal $x(t)$ and output signal $y(t)$ are measured:



The input signal is represented by a dashed line, the output signal by a solid line. Determine the transfer function of the corresponding model.

Solution: The model is determined by using LISA. The numerator order is chosen as $m = 1$, the denominator order as $n = 2$. We get the following result:



The corresponding transfer function is

$$G(s) = \frac{-1.08s + 0.986}{s^2 + 1.777s + 1.025}.$$

Related files: NOISE_X.SIM
NOISE_Y.SIM

 © Ingenieurbüro Dr. Kahlert 1991-2006