6

Simulation and synthesis of state space systems with SUSY

Overview

The WinFACT module SUSY allows the handling with Single input - single output state space systems given as

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{b}u,\; y = \underline{c}^{\mathrm{T}}\underline{x} + du\,,$$

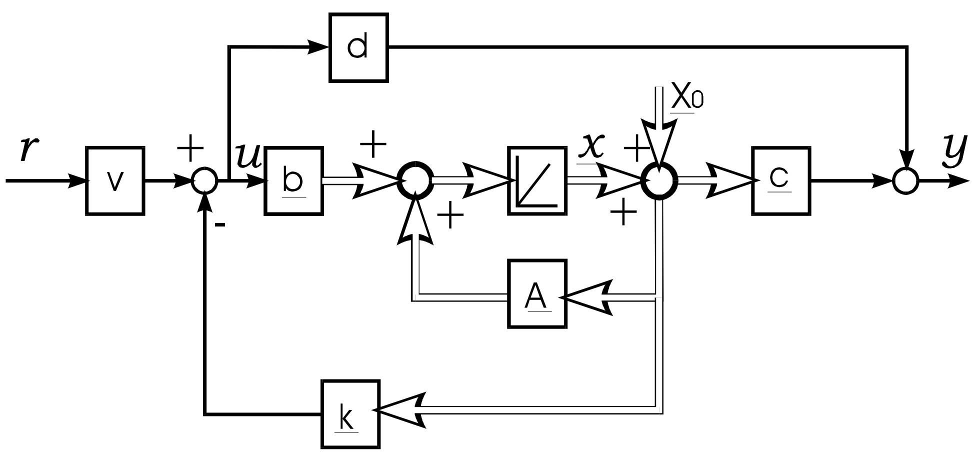
which can be transformed to a closed-loop system by inserting a linear state-feedback controller

$$u = -\underline{k}^{\mathrm{T}}\underline{x} + Vr$$

with

$\underline{A}$	system matrix of open-loop system
$\underline{b}$	input vector
$\underline{c}$	output vector
d	straight-way coefficient
$\underline{x}_0$	system state at inital time $t = 0$
V	precompensator to eliminate a steady-state error
$\underline{k}$	state-feedback vector (controller)

Thus the closed-loop state space system has the following structure:



Closed-loop state space system

SUSY supports your work especially by the following features:

- Easy and comfortable system modification
- Parallel handling of multiple systems
- Representation of simulation results in form of trajectory fields, single trajectories or time responses
- Fast change-over switching between open-loop and closed-loop system
- Eigenvalue calculation for open-loop and closed-loop system
- Controller design by Riccati design, pole placement or by hand
- Document generation (ASCII format)

In the following all features of SUSY are illustrated based on the following sample system:

$$\dot{\underline{x}}(t) = \begin{pmatrix} 0 & -6 \\ -1 & -5 \end{pmatrix} \cdot \underline{x}(t) + \begin{pmatrix} 0 \\ 2 \end{pmatrix} \cdot u(t), \qquad y(t) = \begin{pmatrix} 2 & 0 \end{pmatrix} \cdot \underline{x}(t) + \begin{pmatrix} 0.5 \end{pmatrix} \cdot u(t)$$

System input/output

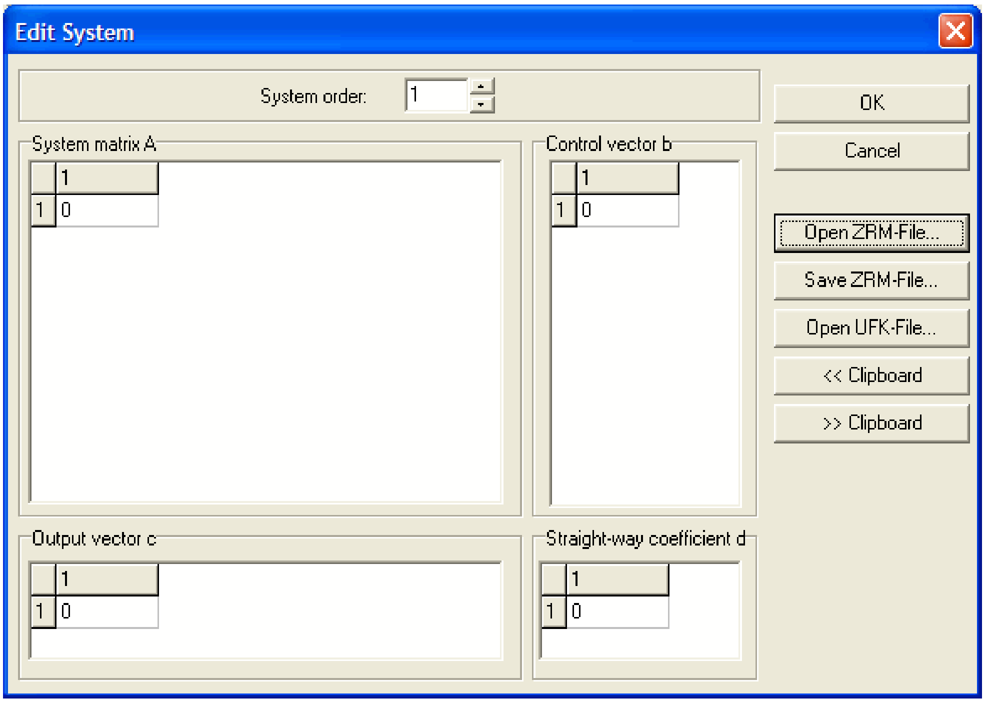
The following chapter lets you get familiar with the different ways to input and modify the system data. This especially includes the manual input and the input by transforming a transfer function. The *System output* chapter describes how to represent all kinds of results in an appropriate form.

System input

Manual input

Before a new system can be specified manually a new system window has to be created by SUSY. This can be done via the FILE | NEW menu option or the corresponding toolbar button. Afterwards the system can be specified via the FILE | EDIT SYSTEM... command and the dialog box shown below. To create our sample system listed above you first have to increase the system order to two. After doing so all input fields are enabled. After having specified the system model you should save it by pressing the *Save ZRM-file...* button. Load and save operations out of this dialog are only related to the state space system itself (i. e. not to the controller), whereas file operations initiated from the program menu (FILE | OPEN SYSTEM FILE... resp. FILE | SAVE SYSTEM FILE (AS)... or the corresponding toolbar buttons also load/save

- the controller design method used at last,
- the parameters of this design method and
- the actual state of the control loop (open or closed).



System modification dialog

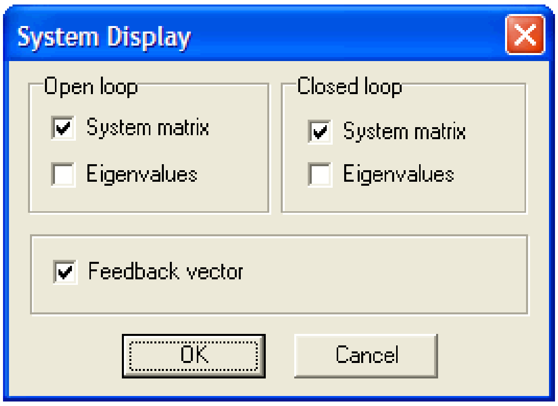
The status bar of the main window displays the current system order.

Loading transfer function

By using the *Open UFK-file...* button of the system modification dialog a transfer function can be loaded from a UFK-file and converted to a state space model in Frobenius-form (for details see [7]).

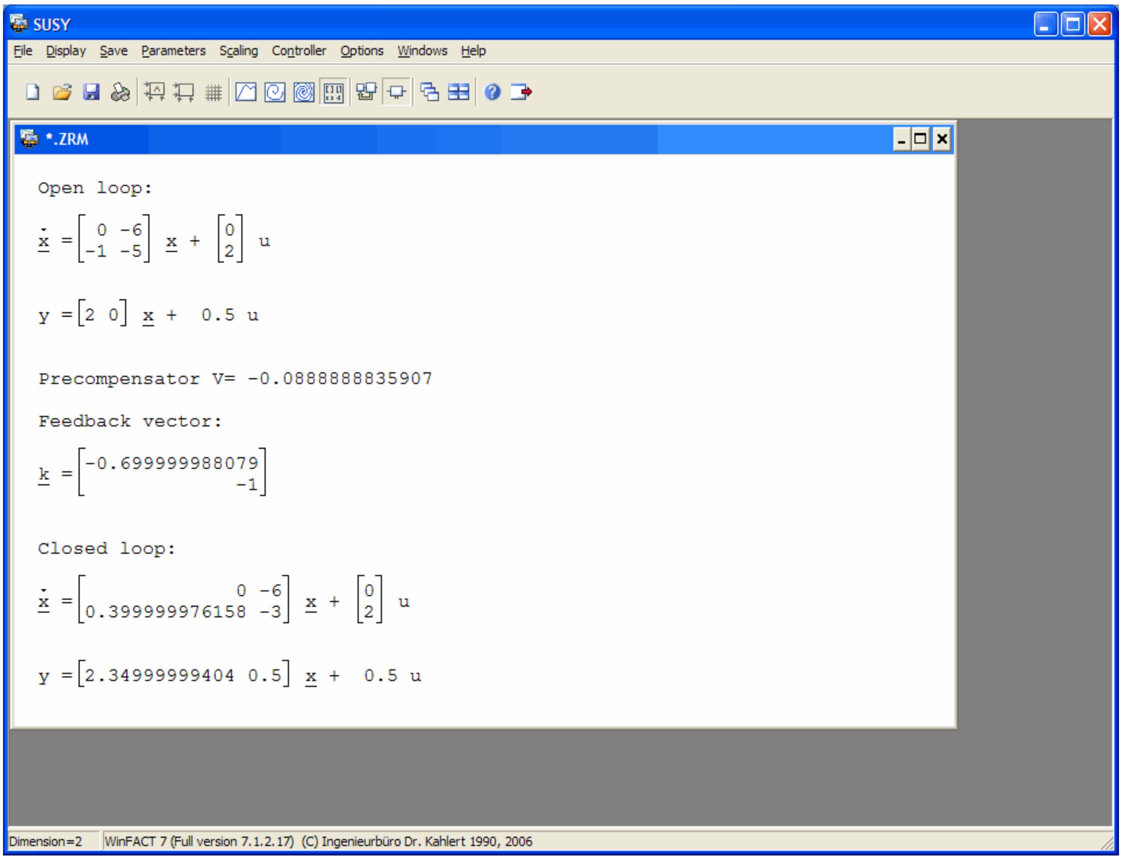
System output

The system output is realized in matrix form. The OPTIONS | SYSTEM DISPLAY... allows the selection of the parameters to be displayed if the output mode is set via DISPLAY | SYSTEM DISPLAY.



Dialog for selection of parameters displayed with system output

Parameters related to the closed-loop system as well as the feedback vector can only be displayed after a controller had been designed.



System display with feedback vector and pre-compensator

Simulation

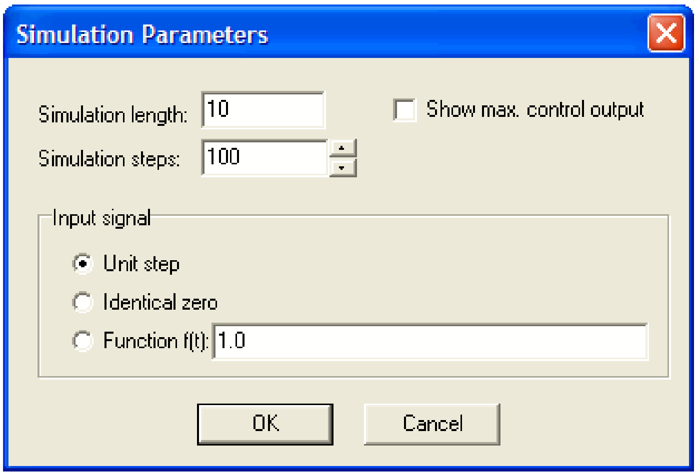
This chapter illustrates the various ways of executing a simulation and presenting the results. You will learn which parameters, options and further settings can be selected.

Simulation parameters

To modify the simulation parameters you have to change to one of the following display modes first:

- Time response
- Single trajectory
- Trajectory field

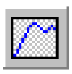
Afterwards the PARAMETERS submenu is enabled. Choose the SIMULATION... menu item to get into the simulation parameters dialog.



Simulation parameters dialog

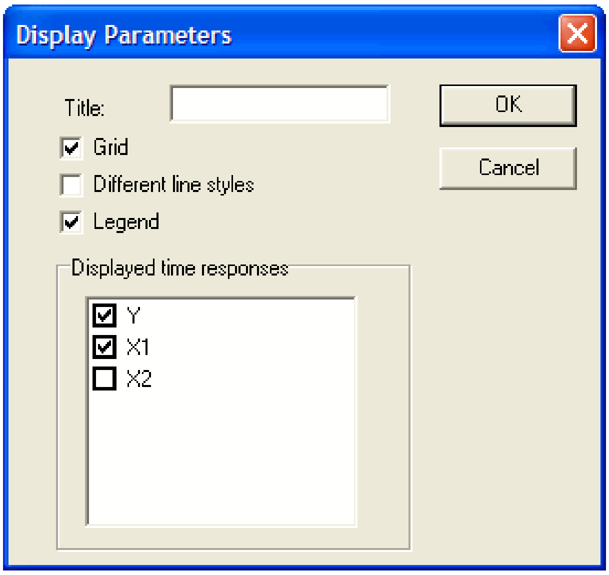
This dialog allows the selection between diverse input functions, the specification of the simulation length and the number of executed simulation steps (and thus the simulation step size). For a detailed description of functions available via the function parser see chapter 10 of this handbook, description of generator-block. If the *Show max. control output* option is activated the maximum value of the manipulated variable $u(t)$ of the closed-loop system is displayed. Please note that the determination of this value can only base on the actually selected simulation length.

Time response

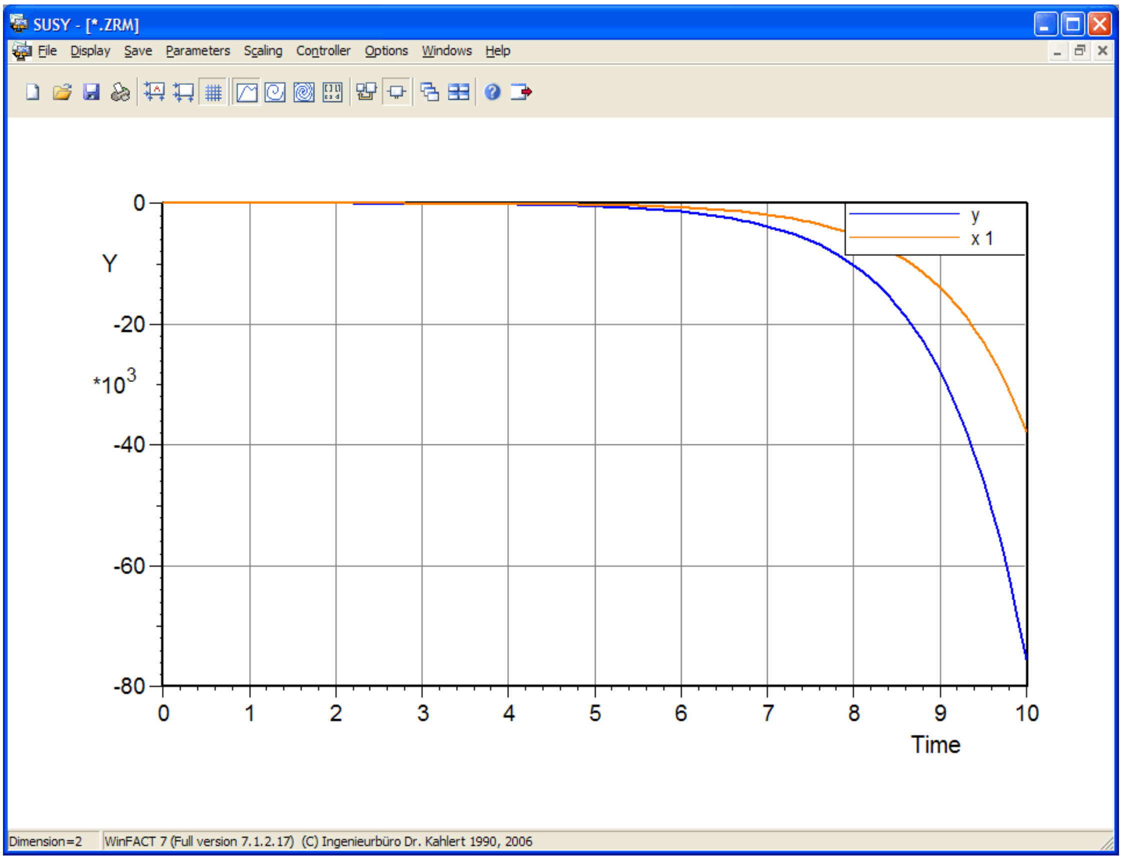


The DISPLAY | TIME RESPONSE menu item or the corresponding toolbar button actiavtes the simulation of the currently configured state space model. Depending on the order of the system not only the output variable y but all state variables can be displayed. To specifiy the variables to be displayed use the PARAMETERS | DISPLAY... menu item.

Because our sample system is of second order the corresponding list of the dialog contains only two state variables (x_1 and x_2). We would like to see the first of these variables, so we activate the corresponding checkbox (see screenshot below). After closing the dialog the selected two variables are displayed.



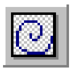
Display parameters dialog



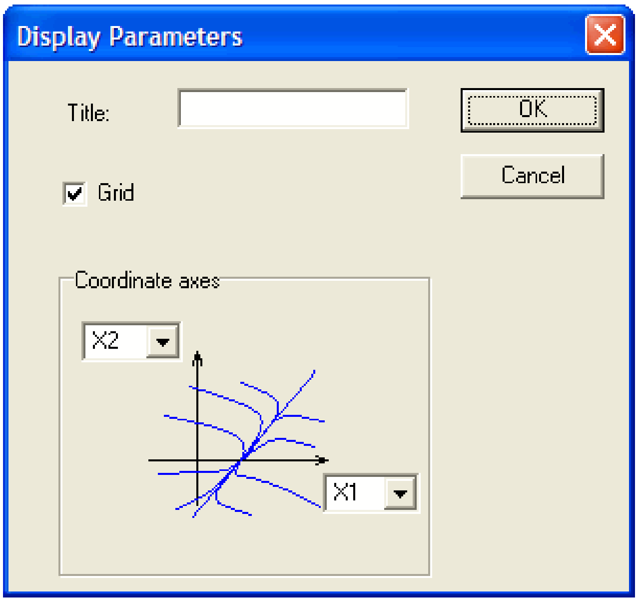
Time response of x_1 and y for sample system

Trajectories

Single trajectories



The DISPLAY | SINGLE TRAJECTORY menu item determines a single trajectory of the system. The PARAMETERS | DISPLAY... option allow the selection of the two state variables the trajectory is based upon before.



Display parameters dialog for single trajectory

Trajectories are drawn in three different colours:

green	Trajektory that starts at the system coordinates at initial time $t = 0$
red	selected trajectory (see *Selecting trajectories*)
blue	all other trajectories

Trajectory fields

The trajectory field display mode is actiavted via the DISPLAY | TRAJECTORY FIELD menu item. Two options are available:

1. THROUGH TRAJECTORY IN T(0)*:* A trajectory is simulated that starts at the coordinates given by state of teh system at the initial time $t = 0$. This trajectory defines the new coordinate system. In the following the start points of the trajectories are chosen in such a way that they are lo-

cated at the borders of this new coordinate system in an appropriate distance. This mode can also be activated via the corresponding toolbar button.

2. FROM CURRENT COORDINATES: The trajectory field is drawn within the current coordinate systems. The start points of the trajectories are located at the borders of this coordinate system.

Selecting trajectories

It can be desirable to select some trajectories e. g. in such cases where the trajectory field is too compact and some of the trajectories are to be deleted. For that purpose just select the trajectories by clicking at them with the left mouse button. All trajectories selected in this way are coloured red.

Deleting trajectories

After having selected trajectories these can be deleted via THE OPTIONS | DELETE TRAJECTORIES menu option.

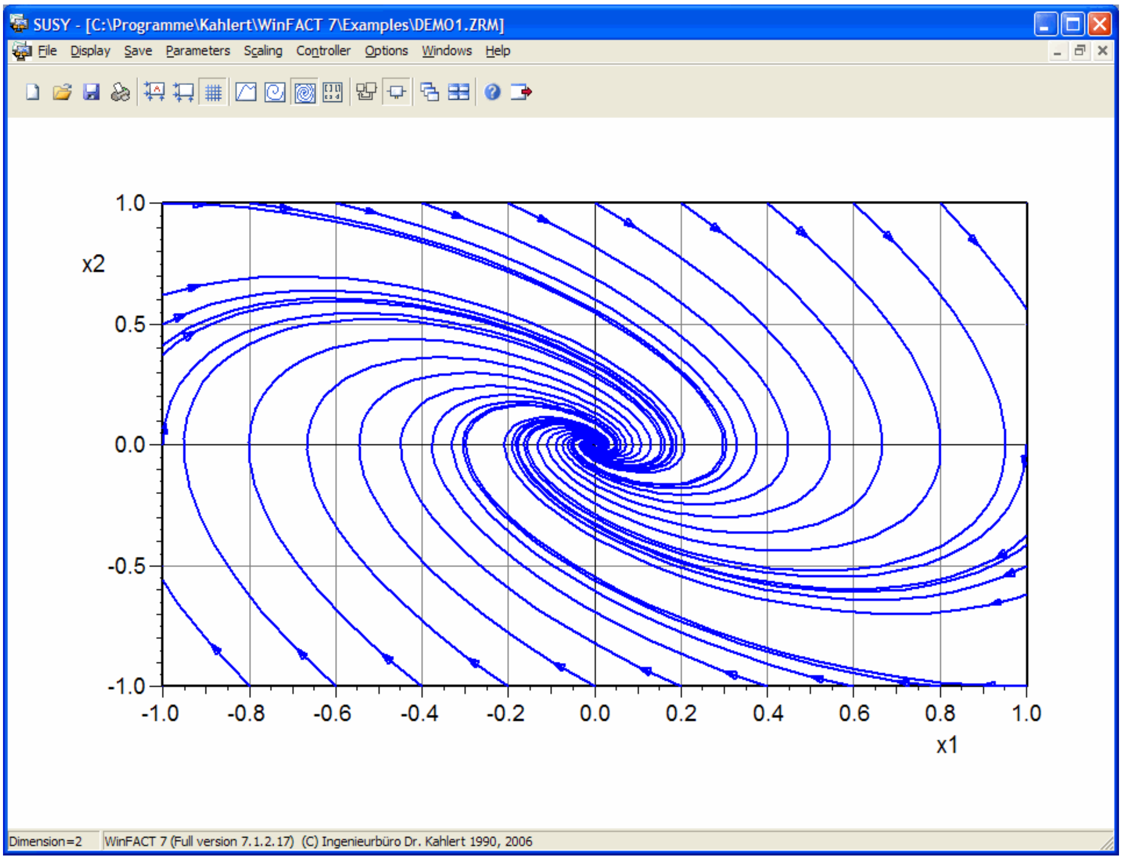
Adding trajectories

A new trajectory can be inserted by a right mouse click within a trajectory field. The current mouse position specifies the start position of the trajectory. The maximum number of trajectories is limited only by the memory of your computer.

Remark: If a new trajectory is inserted in the described way the initial state of the system (system at initial time $t = 0$) is set to the start position of the trajectory.

Direction of trajectories

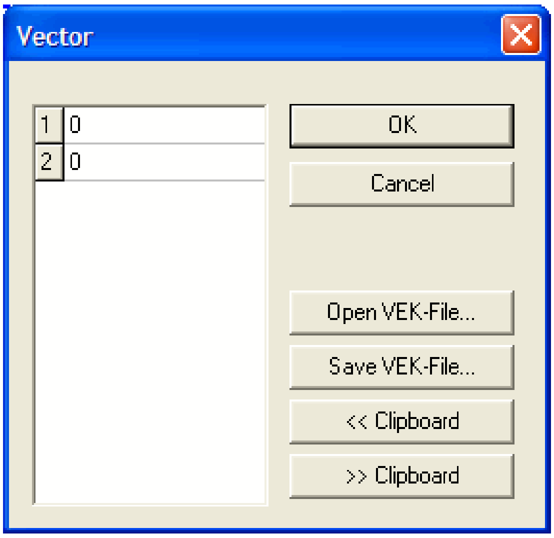
In some cases the direction of trajectories can not clearly be concluded from a trajectory field. SUSY offers a mode where the direction is indicated by an arrow on each trajectory. To activate this mode just select the OPTIONS | SHOW DIRECTION menu option. The screenshot below illustrates this mode.



Trajectory field with integrated direction display via arrows

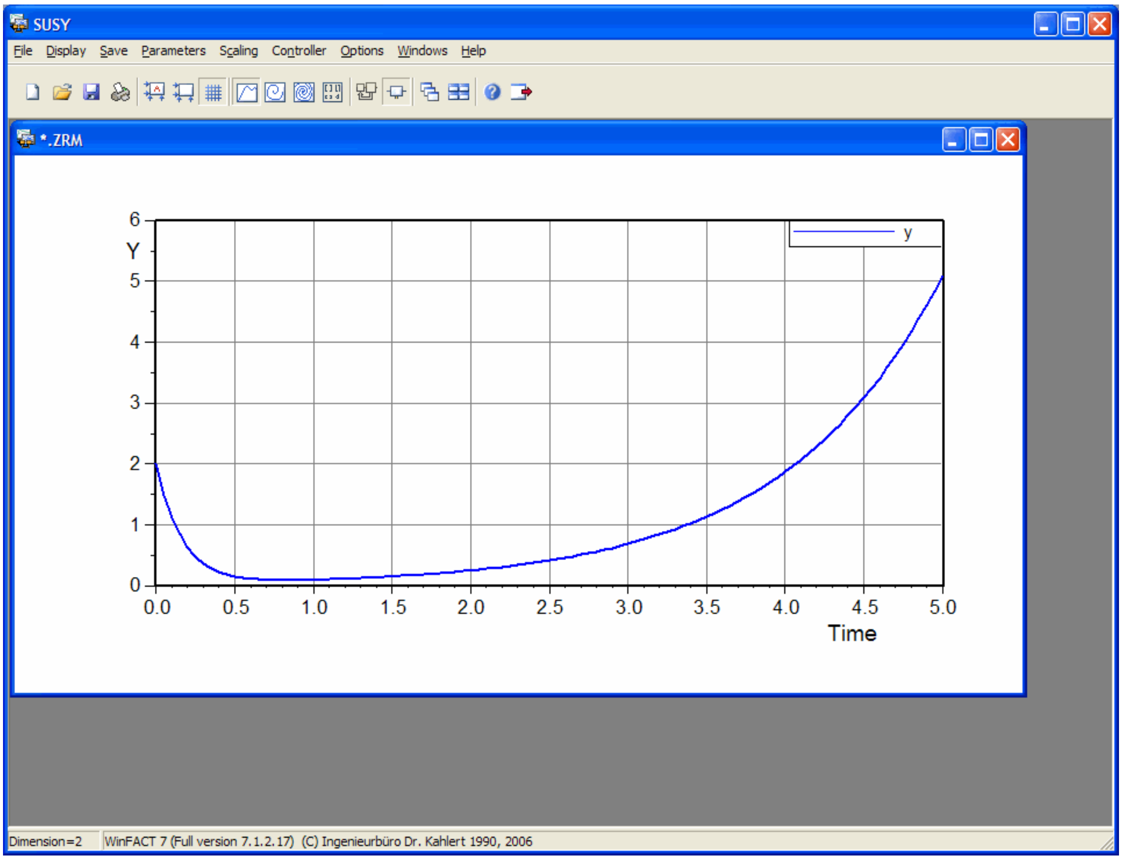
System at time '0'

The system state at time $t = 0$ can differ from $\underline{0}$. If e. g. natural oscillations of a system starting from a specific initial state are to be simulated, you have to set the input function to 0 and specifiy the initial state via the PARAMETERS | SYSTEM AT TIME '0'... menu item (see screenshot below).



Dialog for specification of the initial state

If we select e. g. initial values $x_1(0) = 1$ and $x_2(0) = 0.98$ for our sample system we get the following time response of the output variable:



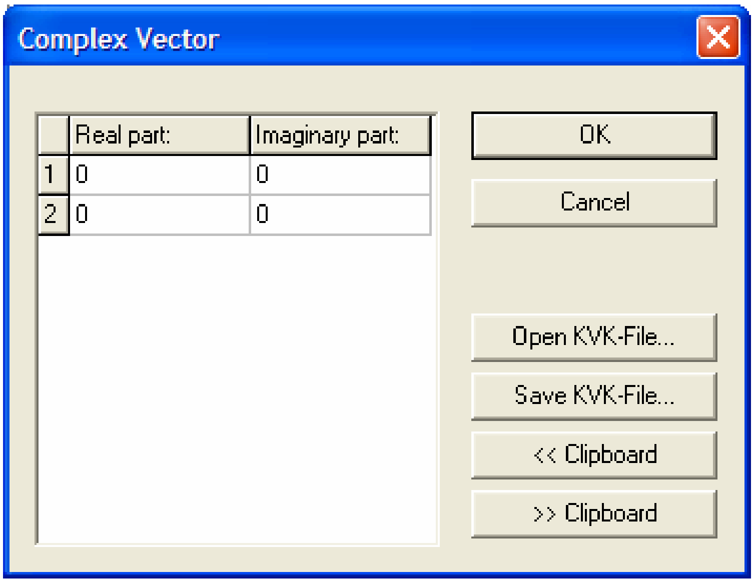
Response of the output variable for a simulation length of 5

Controller design

This chapter shows how to design a linear state feedback controller with SUSY. For that purpose SUSY offers two half-automatical design methods as well as the option to specifiy the controller by hand. The pre-compensator V for the elimination of the steady-state error is calculated by SUSY automatically independent of the selected design method and considered when the simulation results are determined.

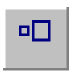
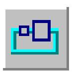
Design by pole placement

The pole placement method is based on a manual specification of the eigenvalues (poles) of the closed-loop system via the CONTROLLER | POLE PLACEMENT | POLE INPUT... menu option which leads to the dialog shown below.



Pole placement dialog

If a complex pole is to be specified also the conjugate complex pole has to be specified; if this pole is missed a error message appears.

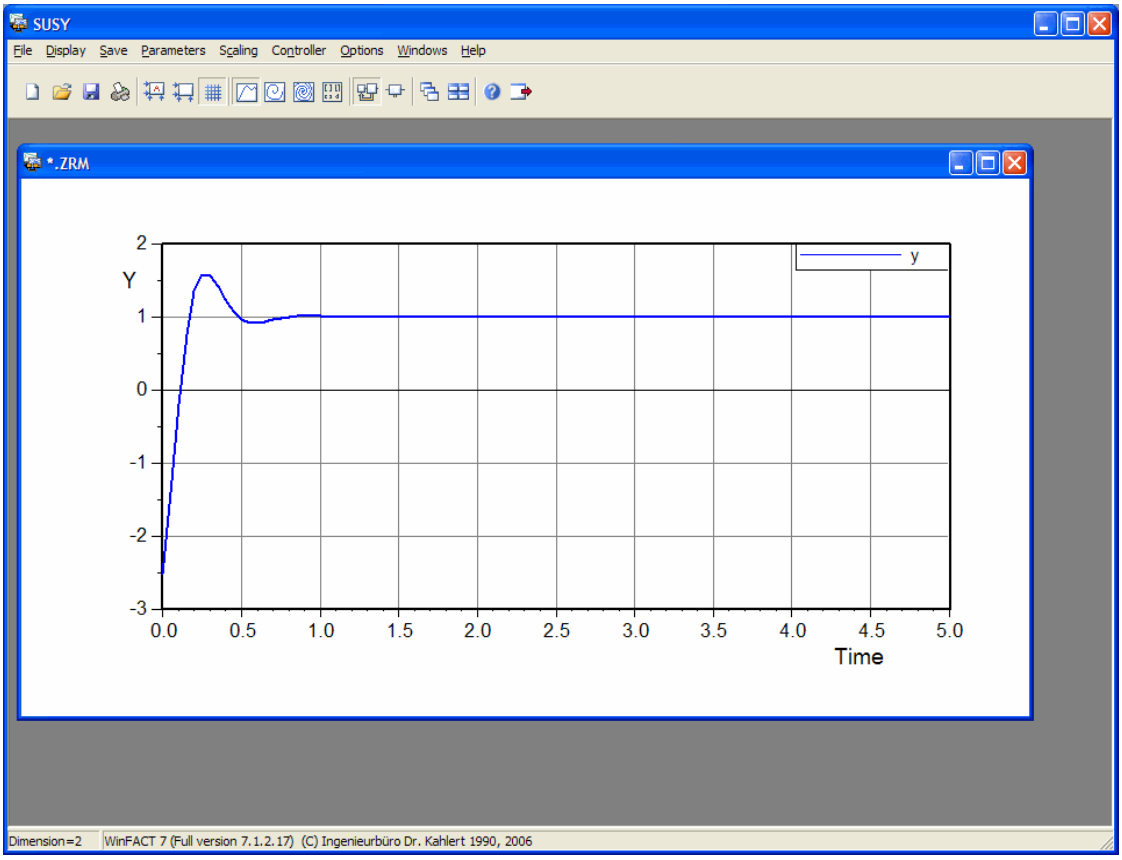
 

After specifiying all poles the CONTROLLER | POLE PLACEMENT | CALCULATE CONTROLLER menu option determines the corresponding state feedback controller. In the following the system display can be switched between open-loop and closed-loop representation by using the corresponding pair of toolbar buttons.

We want do design a controller for our sample system that moves the poles of the closed-loop system to $-6 \pm j10$. We get the following result:

$$\underline{k} = \begin{pmatrix} -11.833 \\ 3.5 \end{pmatrix}.$$

The screenshot below shows the response of the output variable of the closed-loop system to a unit step function of the reference variable.



Step response of output variable of closed-loop system

Riccati design

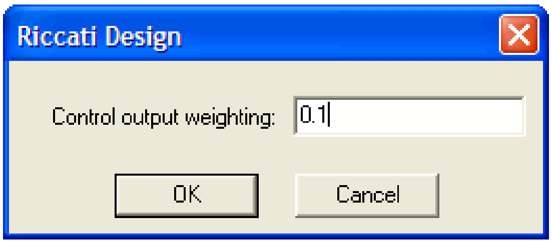
Aim of the Riccati design is the minimization of the performance integral

$$J = \int_0^\infty \underline{x}^{\mathrm{T}} \underline{Q}\,\underline{x} + \beta u^2 \,\mathrm{d}t$$

with

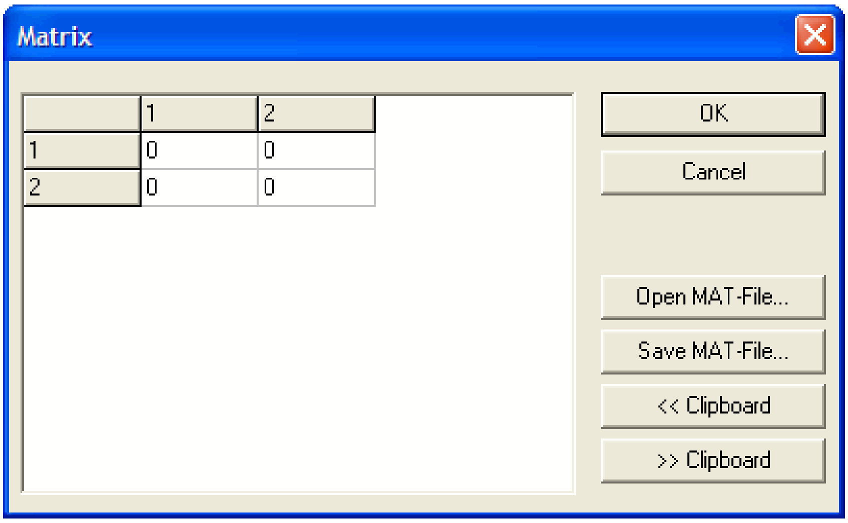
u	manipulated variable (control output)
$\underline{x}$	state vector
$\underline{Q}$	weighting matrix for state variables
β	weighting coefficient for manipulated variable

The controller is determined by an iterative numerical solution of the complete Riccati equation. The parameters β and $\underline{Q}$ have to be specified by the user. The weighting coefficient β is specified via the CONTROLLER | RICCATI DESIGN | CONTROL OUTPUT WEIGHTING... menu option. A larger value of β leads to a "weaker" controller, i. e. a controller that generates lower values for the manipulated variable *u*.



Dialog for weighting coefficient β

The weighting matrix $\underline{Q}$ is specified via the CONTROLLER | RICCATI DESIGN | MATRIX Q... menu option. The matrix dimension is equivalent to the actual system order, thus it cannot be changed.

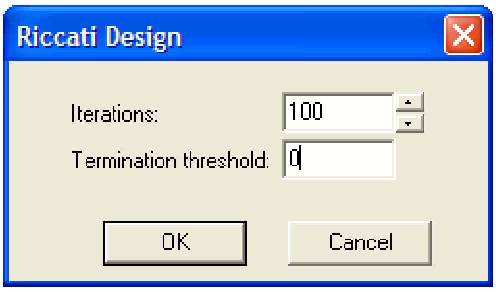


Input of weighting matrix Q

Last but not least two parameters have to be specified which influence the numerical algorithm used to solve the Riccati equation. These are

- the maximum number of iterations
- the termination threshold

The default values should be changed only in case of convergence problems.



Riccati parameters

We want to design a Riccati controller for our sample system based on the following design parameters:

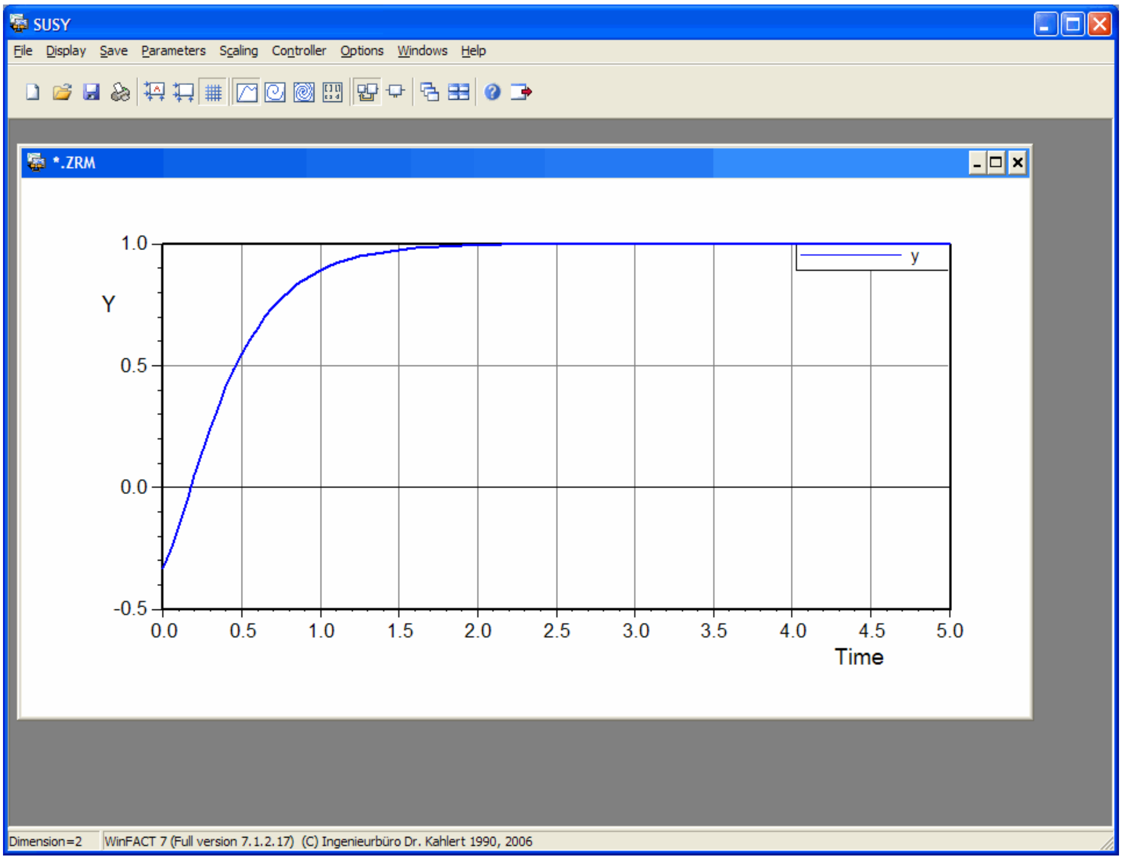
- $\underline{Q} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- $\beta = 0.5$
- Iterations: 100

- Termination threshold: 0.00001

The design leads to the controller

$$\underline{k} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

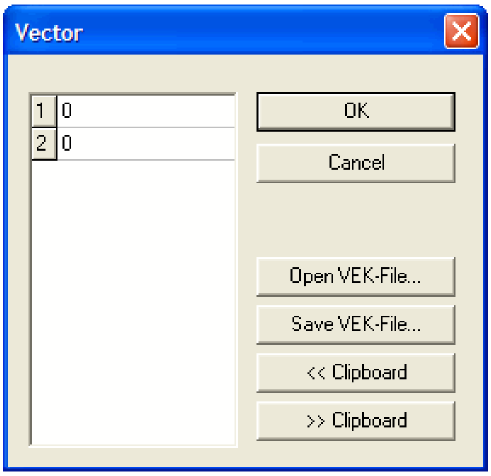
and the closed-loop step response shown in the screenshot below.



Step response of output variable of closed-loop system

Manual controller input

The CONTROLLER | EDIT CONTROLLER... menu options allows the user to specifiy or modify the controller by hand.

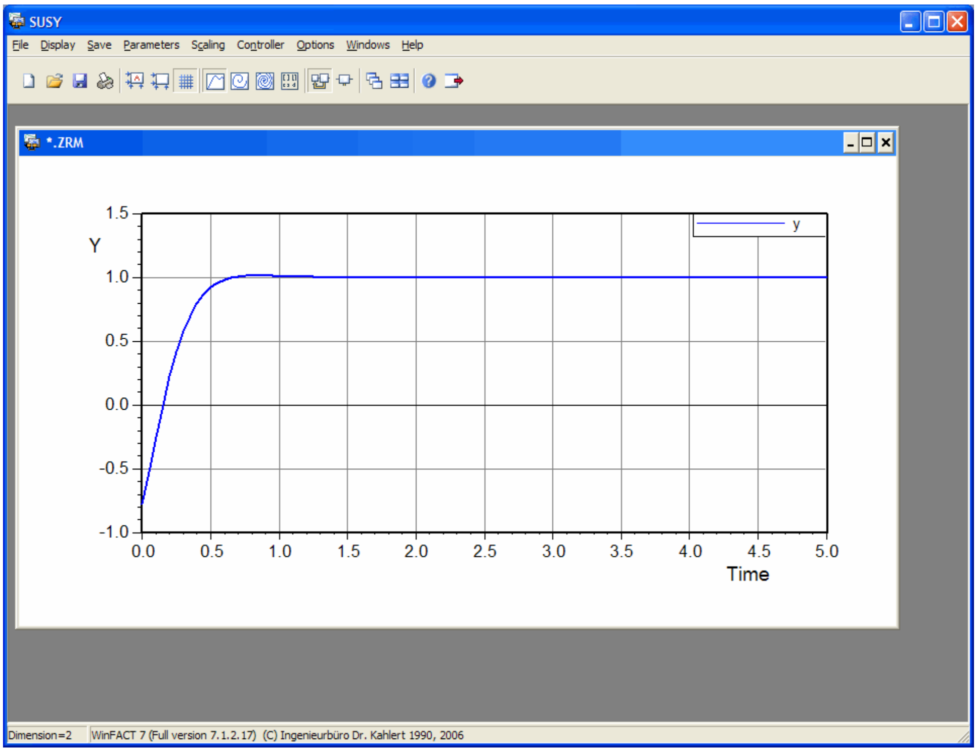


Manual controller specification

If we e. g. change the controller designed by Riccati above to

$$\underline{k} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

and leave the dialog via the OK button the closed-loop system is automatically updated based on the new controller. The simulation yields the following results:



Administrative operations

This chapter deals with the generation of documents as well as with ways of controlling the display.

Text documents

Text documents can be created by selecting the SAVE | SAVE... menu option while the system display mode is active. After specifiying the output file name the following parameters are saved in the corresponding ASCII file:

- The open-loop system including its eigenvalues
- The closed-loop system including its eigenvalues (only if a controller was designed earlier)
- All design parameters (depending on the selected design method)
- The state feedback controller

The listing below gives an example for such a document file after having designed a Riccati controller.

```
        WinFACT - Windows Fuzzy And Control Tools
           (C) Ingenieurbuero Dr. Kahlert 1998

Document file for: C:\tmp\test.doc
created at: 22.09.98 12:21:26

------------------------------------------------------
State space model: open

System matrix A:
              0              -6
             -1              -5

Input vector b:
```

```
              0
              2

Output vector c:
              2              0

Straight-way coefficient d:
            0.5

Eigenvalues of open-loop system:
Real part:           Imaginary part:
             -6                   0
              1                   0
-------------------------------------------------------
State space model: closed

System matrix A:
              0                  -6
   2.99999952316       -8.99999904633

Input vector b:
              0
              2

Output vector c:
   2.99999976158      -0.999999880791

Straight-way coefficient d:
            0.5

Eigenvalues of closed-loop system:
Real part:           Imaginary part:
  -6.00000000003                  0
  -2.99999946895                  0
-------------------------------------------------------
Weighting matrix Q:
              1                   0
              0                   1

Weighting coefficient Beta:
0.5
```



```
State feedback vector:
  -1.99999976158
   1.99999976158

Pre-compensator V=  -0.666666548657
```

Saving curves

Saving time responses

If the time response display mode is active THE SAVE | SAVE... menu option allows saving all displayed curves

- as a SIM-file if only one curve is displayed,
- as a MXY-file if more than one curve is displayed.

The corresponding file formats are described in detail in chapter 2 *Basics*.

Saving trajectories

If the active SUSY window contains trajectories you have to select those trajectories you want to save first (see chapter *Selecting trajectories*). Afterwards the SAVE | SAVE... menu option allows saving of the selected curves as a MXY-file.